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Mazzoleni - Regularity of the minima of integral shape functionals: the non-degenerate case

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Abstract. We prove the rst regularity theorem for the free boundary of solutions to shape optimization problems with integral functionals, for which the energy of a domain
is obtained as the integral of a cost function j(u; x) depending on the solution u of a certain PDE problem on Omega. The main feature of these functionals is that (in general) the minimality of a domain cannot be translated into a variational problem for a single (real or vector valued) state function, which is the starting point in all the previous regularity results for optimal shapes.
In this paper the focus on the case of a ne cost functions j(u; x) =-g(x)u + Q(x), where u
is the solution of the PDE -laplaciano u = f with Dirichlet boundary conditions. We obtain the Lipschitz continuity and the non-degeneracy of the optimal u from the inwards/outwards optimality of and then we use the stability of with respect to variations with smooth vector elds in order to study the blow-up limits of the state function u. By performing a triple consecutive blow-up, we prove the existence of blow-up sequences converging to homogeneous stable solution of the one-phase Bernoulli problem and according to the blow-up limits, we decompose boundary of Omega into a singular and a regular part. In order to estimate the Hausdorff dimension of the singular set of boundary of Omega we give a new formulation of the notion of stability for the one-phase problem, which is preserved under blow-up limits and allows to develop a dimension reduction principle. Finally, by combining a higher order Boundary Harnack principle and a viscosity approach, we prove C1 regularity of the regular part of the free boundary
(if the data are smooth).
This is a joint work with Giuseppe Buttazzo, Francesco P. Maiale, Giorgio Tortone and Bozhidar Velichkov (Pisa).
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