Ricercatori in Algebra e Geometria 2023

Wednesday, 20 September 2023 - Friday, 22 September 2023

Argiletum

Book of Abstracts

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Künneth decompositions of algebraic varieties

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Given a nice topological space X, for example a topological manifold, one can study it by means of a cellular decomposition, which allows to define and compute important invariants of X. But even when X arises from a smooth projective algebraic variety, cellular decompositions may not be of an algebraic nature. By replacing them with the notion of "Künneth decompositions", one hopes to provide a purely algebraic way of defining the resulting invariants. The aim of this talk is to give an overview of these ideas and to explain recent progress on the existence of Künneth decompositions, in particular for hyperkähler varieties.

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Algebraic cycles, p-adic L-functions, and the oscillator representations

Author: Daniel Disegni[1]

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A classical arithmetic problem is that of finding and classifying rational solutions (or families thereof) to systems of polynomials equations. If one linearizes it to allow combinations of solutions (algebraic cycles), the problem can be decomposed based on 'motives'. I will introduce some 'canonical' algebraic cycles for high rank motives with a similar symmetry to elliptic curves.
The cycles arise from Shimura varieties via the oscillator representation of Segal and Weil. Under some conditions, one can relate their nontriviality to L-functions and show that, if nontrivial, there are 'no other cycles'. This supports analogues of the Birch and Swinnerton-Dyer conjecture. (Partly based on joint work with Yifeng Liu.)

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On the intersection cohomology of vector bundles

Author: Camilla Felisetti[1]

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The moduli space of vector bundles on curves is a classical object in modern algebraic geometry. Mumford proved that it is a projective, generally singular variety, thus it is natural to investigate its topological properties, such as the cohomology. In this talk we study a natural invariant which is well behaved also for singular varieties, namely intersection cohomology. The study of the intersection cohomology of the moduli spaces of semistable bundles on curves began in the 80's with the works of Kirwan. Motivated by the work of Mozgovoy and Reineke, in joint work with Andras Szenes and Olga Trapeznikova, we give a complete description of the intersection cohomology of vector bundles via a detailed analysis of the celebrated Decomposition Theorem of Beilinson, Bernstein, Deligne and Gabber applied to a certain map from parabolic bundles. We also give a new formula for the intersection Betti numbers of these moduli spaces, which has a clear geometric meaning.

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Abelian and cyclotomic points in backward orbits of rational functions

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Given a rational function ϕ in one variable and a point α of the projective line over a number field K, one can ask how many points in the backward orbit of α belong to a cyclotomic, or more in general abelian, extension of K. Conjecturally, there should be only finitely many such points, unless the map ϕ is of a very special form. In this talk, I will explain how one can approach this problem via two very different techniques, one relying on unlikely intersection methods and the other on group theoretical arguments. The combination of these techniques allows to prove several cases of the aforementioned conjecture. This is based on joint works with Pagano and Ostafe-Zannier.

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Spectral theory of isogeny graphs

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A familiar object in isogeny based cryptography is the graph whose vertices are supersingular elliptic curves and whose edges are isogenies of fixed degree ℓ. It is immediate to prove that from each vertex there are exactly ℓ+1 outgoing edges, while it is less obvious that such a graph is connected and that it has the Ramanujan property, a property about the spectrum of the adjacency matrix implying that random walks very soon visit all vertices with the same probability. In our talk we look at a generalization of these graphs, namely graphs whose vertices are pairs (E,T), where E is a supersingular elliptic curve and T is some information on the n-torsion of E (e.g. a basis, a point, a subgroup) for fixed n. These graphs can be multipartite, implying that the Ramanujan property is not always satisfied. By studying modular curves over mixed characteristic we relate isogeny graphs to geometric and cohomological objects, which allows us to prove the appropriate modification of the Ramanujan property.

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Higher Fano manifolds

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Fano manifolds are complex projective manifolds having positive first Chern class. The positivity condition on the first Chern class has far-reaching geometric and arithmetic implications. For instance, Fano manifolds are covered by rational curves, and families of Fano manifolds over one-dimensional bases always admit holomorphic sections.

In recent years, there has been a great effort toward defining suitable higher analogues of the Fano condition, which are expected to enjoy stronger versions of several of the nice properties of Fano manifolds. In this talk, I will discuss a possible notion of higher Fano manifolds in terms of the positivity of higher Chern characters and discuss geometric features of these manifolds.

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On the first Hochschild cohomology

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Let A be an associative algebra. The Hochschild cohomology of A has a rich structure: it is a Gerstenhaber algebra. In particular, its first degree component, denoted by $\mathrm{HH}^1(A)$ is a Lie algebra. In positive characteristic $\mathrm{HH}^1(A)$ is a restricted Lie algebra.

In the first part of this talk, I will show the invariance, as a restricted Lie algebra, of the first Hochschild cohomology under derived equivalences and under stable equivalences of Morita type for symmetric algebras.

In the second part, I will focus on the relation between the fundamental groups associated to presentations of A and the maximal tori in $\mathrm{HH}^1(A)$. As an application, I will show that if two finite dimensional monomial algebras are derived equivalent, then their Gabriel quivers contain the same number of arrows. For gentle algebras, this was proven by Avella-Alaminos and Geiss.

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The non-degeneracy invariant of Enriques surfaces: a computational approach

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For an Enriques surface S, the non-degeneracy invariant nd(S) retains information about the elliptic fibrations of S and its projective realizations. While this invariant is well understood for general Enriques surfaces, it becomes challenging to compute when specializing our Enriques surface. In this talk, we introduce a combinatorial version of the non-degeneracy invariant that depends on S along with a configuration of smooth rational curves, and gives a lower bound for nd(S). We also provide a SageMath code that computes this combinatorial invariant and we apply it in several examples where nd(S) was previously unknown. This is joint work with Riccardo Moschetti and Franco Rota.

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The (logarithmic) classification problem

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I will give an overview of the classification problems in algebraic geometry, with special attention to the classification of irregular varieties with small invariant. Then I will turn the attention to open varieties and expalin how the classification problem can be generalized in this setting. I will conclude presenting the recent characterization of semi-abelian varieties, which is a joint work with R. Pardini and M. Mendes Lopes.

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Rational cohomology of moduli spaces of curves

Author: Angelina Zheng[1]

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In this talk we will discuss two different methods to study the rational cohomology of moduli spaces of curves. The first one is Gorinov-Vassiliev's method, which is a topological method that computes the cohomology of complements of discriminants. Similar information on the cohomology of a moduli space can be obtained through a different method, namely by counting its points over finite fields. The two methods will be presented by going through a basic example. Various results on the cohomology of moduli spaces of curves obtained using such methods will also be mentioned.