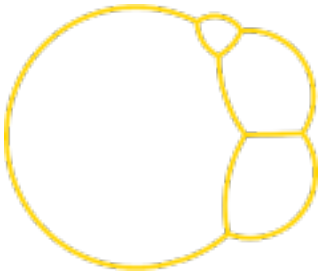

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Estimates on the Cheeger constant

Monday, 20 June 2022 17:00 (50 minutes)

Given a set $\Omega \subset \mathbb{R}^N$, the Cheeger constant is a purely geometrical quantity defined as the infimum $\mathrm{h}(\Omega) := \inf \left\{ \frac{P(E)}{|E|} \ : \ E \subset \Omega, \, |E| > 0 \right\}$.

Despite seeming unassuming, it pops up in many contexts that apparently have nothing in common. To name a few, under some mild regularity assumptions on Ω: bounds on the first Dirichlet eigenvalue of the p-Laplacian; existence of sets in Ω or of graphs over Ω with prescribed curvature; threshold of vertical load that a flat membrane can sustain before breaking; image reconstruction and denoising. The constant of the unit square has even been a tool in a elementary proof of the Prime Number Theorem!

Given the numerous applications, it is important being able to explicitly compute the constant. This is in general a hard task: a telltale sign is that we do not know the exact value of the constant of the unit cube in dimension $N \geq 3$. The computation is (theoretically) feasible for a large class of Jordan domains in the plane [LNS] or in very special cases in general dimension.

If unable to compute the constant, it would be at least desirable to obtain bounds on it: in [LNS] we proved bounds via interior approximations of the set for 2d domains on which, at least on a theoretical level, the constant can be found by solving an algebraic equation; in [JS] a quantitative inequality for the Cheeger constant has been proved in terms of the Riesz asymmetry; in [BPS] bounds of the constant for cylindrical domains $\Omega = \omega \times [0, L]$ have been shown in terms of the constant of the cross-section ω.

[BPS] G. Buttazzo, A. Pratelli, and G. Saracco. Upper and lower bounds on the first Dirichlet eigenvalue of the p-Laplacian in cylindrical domains, and existence of minimizers of a shape optimization problem. Forthcoming.

[JS] V. Julin and G. Saracco. "Quantitative lower bounds to the Euclidean and the Gaussian Cheeger constants."In: Ann. Fenn. Math. 46.2 (2021), pp. 1071–1087.

[LNS] G. P. Leonardi, R. Neumayer, and G. Saracco. "The Cheeger constant of a Jordan domain without necks."In: Calc. Var. Partial Differential Equations 56.6 (2017), p. 164.

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