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Nonstandard methods without the axiom of choice

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Model-theoretic frameworks for nonstandard methods entail the existence of nonprincipal ultrafilters over $\mathbb{N}$, a strong version of the Axiom of Choice (**AC**). While **AC** is instrumental in many abstract areas of mathematics, such as general topology or functional analysis, its use in infinitesimal calculus or number theory should not be necessary.
In [1], Mikhail Katz and I have formulated a set theory **SPOT** in the **st**-$\in$-language. The theory has three simple axioms, Transfer, Nontriviality and Standard Part. It is a subtheory of the nonstandard set theories **IST** and **HST**, but unlike them, it is a conservative extension of **ZF**. Arguments carried out in **SPOT** thus do not depend on any form of **AC**. Infinitesimal calculus can be developed in **SPOT** as far as the global version of Peano's Theorem (the usual proofs of which use **ADC**, the Axiom of Dependent Choice). The existence of upper Banach densities can be proved in **SPOT**. The conservativity of **SPOT** over **ZF** is established by a construction that combines and extends the methods of forcing developed by A. Enayat and M. Spector.
A stronger theory **SCOT** is a conservative extension of **ZF** + **ADC**. It is suitable for handling such features as an infinitesimal approach to the Lebesgue measure.
I will also explore the possibilities for extending these theories to multiple levels of standardness and the relevance of such extensions to infinitesimal calculus and R. Jin's proof of Szemerédi's Theorem.

[1] K. Hrbacek and M. G. Katz, Infinitesimal analysis without the Axiom of Choice, Ann. Pure Appl. Logic 172, 6 (2021).
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