

The complexity of proving Ramsey principles

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We say that a graph with n vertices is c -Ramsey if there is no set of $c \log n$ vertices which form in G either a clique or an independent set. In other words a c -Ramsey graph over n nodes is a witness of the fact that $r(c \log n) > n$, where $r(k)$ is the least N such that every graph of size at least N contains a clique or independent set of size k .

In searching for hard formulas to prove, proof complexity has often focused on Ramsey principles, by studying upper bounds for $r(k)$, that is on the complexity of certifying that a graph G of size n is c -Ramsey.

After reviewing what is known about the complexity of proving Ramsey principles we discuss the case of the k -clique Principle, an instantiation of Ramsey principles stating that a graph on n vertices does not contain a k -clique. We will present a recent result of ours on the complexity of proving such principle in systems related to the Resolution and when the graph G is a random graph and we discuss why its exact proof complexity in Resolution represents a major open problem.

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