

Initial Applications of Alpha Theory in Telecommunication Engineering

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Heavy-tailed distributions with infinite variance are crucial in telecommunication engineering as they can model real-world phenomena accurately. They describe various traffic characteristics, such as file sizes on web servers and corresponding transmission times, CPU times, idle times, peak rates and connection times. However, simulating these distributions presents numerical challenges due to infinite variance and slow convergence. To address these issues, we developed a Matlab toolbox based on the Alpha Theory developed by Benci and Di Nasso (and the associated concept of Euclidean numbers). This approach allows for a new Euclidean Heavy-Tailed Lognormal distribution with finite mean and numerically verifiable infinite variance, recently appeared in [1]. Finally we will discuss its use in Queueing Theory, where numerous estimation and performance prediction formulations for queues have inherent mathematical limitations when considering infinite or infinitesimal values. As an example, we will show how the Euclidean LogNormal distribution could be used in combination with the Pollaczek-Khinchin formula in M/G/1 queues, even when the variance is infinite, provided that it is a computable value (and not a value diverging towards $+\infty$). We expect that this will also allow us to perform asymptotic analyses numerically in Matlab (instead of using paper and pencil or symbolic software like Mathematica).

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[1] Cococcioni, M., Fiorini, F. and Pagano, M. (2023) "Modelling Heavy Tailed Phenomena Using a LogNormal Distribution Having a Numerically Verifiable Infinite Variance", *Mathematics*, 11(7):1758, <https://doi.org/10.3390/math11071758>

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