Enumerative Geometry and Geometric Representation Theory

Monday, June 23, 2025 - Friday, July 4, 2025

Università di Pisa, Italy

Scientific Program

First Week

Joel Kamnitzer: *From quantum cohomology of symplectic resolutions to compactifications of families of Gaudin algebras*

Abstract: In recent years, symplectic resolutions have been a highly active area of study in algebraic geometry and representation theory. I will introduce this area, including the quantum cohomology of symplectic resolutions. Quantum cohomology will naturally lead us toward the theory of quantum integrable systems, which are large commutative subalgebras inside non-commutative algebras. In particular, we will consider Bethe subalgebras in Yangians, and Gaudin algebras inside tensor products of universal enveloping algebras. These algebras are related (respectively) to the quantum cohomology of quiver varieties and affine Grassmannian slices. Finally, we will switch gears and study the compactification of the parameter space of these algebras. Remarkably, this will lead us to the definition of new moduli spaces of curves.

Dinakar Muthiah's Exercise Sessions: *The BFN construction of Coulomb branches, symplectic duality, and affine Grassmannians slices*

Abstract: I will discuss the Braverman-Finkelberg-Nakajima construction of Coulomb branches. Their construction gives rise to many examples of symplectic duality. In particular, their construction gives rigorous meaning to the statement that finite-type Nakajima quiver varieties are symplectically dual to affine Grassmannian slices.

Dinakar Muthiah's Research Session: *Kac-Moody affine Grassmannian slices*

Abstract: I will explain how to use the BFN construction to define Kac-Moody affine Grassmannian slices, which allow us to approach the Geometric Satake Correspondence in Kac-Moody types. This final talk will include discussion of joint work with Alex Weekes.

Bernd Siebert: *Logarithmic Gromov-Witten invariants and its application to mirror symmetry*

Abstract: The purpose of this lecture series is a gentle introduction to the subject of logarithmic Gromov-Witten invariants and its application to mirror symmetry. Logarithmic geometry in the sense of Illusie and Kato is a version of algebraic geometry adapted to situations relative a divisor with normal or even toroidal crossings, including central fibers of normal crossing degenerations. Logarithmic Gromov-Witten theory enhances Gromov-Witten theory with contact conditions with these divisors, even when the curve has components mapping into the divisor. Apart from a richer theory coming from the added information, such contact conditions naturally come up in degeneration and gluing situations.

A remarkable feature of the theory is that the usual dual intersection graphs of the domain curves arising in ordinary Gromov-Witten theory become promoted to tropical curves. In Calabi-Yau situations arising in mirror symmetry, these tropical curves sweep out the wall structures carrying the quantum corrections in the construction of mirror geometries.

Lectures:

I. Geometric introduction to logarithmic geometry

II. Kato-Nakayama spaces of log spaces, tropicalization, stable log maps

III. Logarithmic Gromov-Witten invariants, Artin fans, punctured Gromov-Witten invariants

IV. The gluing formalism for log Gromov-Witten theory

V. Intrinsic mirror symmetry

Second Week

Junliang Shen: *Decomposition Theorem for abelian fibrations*

Abstract: The decomposition theorem of Beilinson, Bernstein, Deligne, and Gabber allows us to compute the “relative cohomology” of a proper map in terms of “simple objects”; however, the “simple objects” are not really simple in general, so it is very difficult to carry out explicit calculations. In his proof of the fundamental lemma of the Langlands program, Ngô initiated a systematic study of the decomposition theorem for abelian fibrations (i.e. a proper map whose general fibers are abelian varieties and special fibers are their degenerations); this package is nowadays referred to as the Ngô support theorem. In the last two decades, these techniques have been further developed by many people, and have rich applications in several directions, including the topology of Hitchin systems, BPS invariants in enumerative geometry, the study of compact hyper-Kähler manifolds, algebraic cycles etc. In the lectures, I will start with the decomposition theorem, then I will discuss ideas of the Ngô support theorem, and present some recent applications.

Olivier Schiffmann: *Cohomological Hall algebras of sheaves on surfaces and applications*

Abstract: The aim of these lectures is to explain the construction of an associative algebra structure on the Borel-Moore homology of the stack of properly supported coherent sheaves on a smooth complex surface (the COHA) and to give some motivations and applications. More precisely, we hope to touch upon the following topics:

Definition of the COHA of the preprojective algebra of a quiver, and the relation to Kac-Moody (and larger!) algebras and Kac polynomials.

Definition of the COHA of zero-dimensional sheaves on S, and computation of that COHA (at least under some assumption on S, for instance if S is projective). Relation to W-algebras.

Action of the above COHA of zero-dimensional sheaves on suitable moduli stacks / spaces of coherent sheaves on S. Examples related to Hilbert schemes on S, instanton spaces for the special case $S=\mathbb{A}^2$, moduli stacks of Higgs sheaves for $S=T^*C$, C a smooth projective complex curve.

Definition of the COHA for arbitrary (properly supported) coherent sheaves on a surface S, and some examples / applications (depending on time)