

Sato-Tate groups of Fermat Jacobians

Friday, 7 June 2024 16:00 (1 hour)

The arithmetic of abelian varieties is often studied through the lens of their Galois representations. Given an abelian variety A over a number field K , an important invariant is the so-called Sato-Tate group $operatorname{ST}(A)$, a compact Lie group which conjecturally describes the asymptotic distribution of the characteristic polynomials of Frobenius acting on the Tate modules of A/K . The group of connected components of

$operatorname{ST}(A)$ has particular arithmetic significance, and there exists a unique minimal extension L/K such that

$operatorname{ST}(A_L)$ is connected. There is currently no general technique to determine $operatorname{ST}(A)$, nor the extension L/K . In this talk I will describe how to compute these two invariants for the Jacobian of the curve $y^2 = x^m + 1$ by relating them to the cohomology of (several) Fermat hypersurfaces $X_m^n : Y_0^m + \cdots + Y_{n+1}^m = 0$. The structure of this cohomology has been studied extensively by Deligne; our application, however, will require a more detailed analysis of the action of the absolute Galois group of \mathbb{Q} on the étale cohomology of X_m^n .

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