

# **Regulators V**

**Monday, 3 June 2024 - Thursday, 13 June 2024**

**Department of Mathematics, University of Pisa**

## **Programme**

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# Monday, 3 June 2024

## Higher regulators of the ring of integers (10:00 - 11:00)

- **Presenter: BROWN, Francis (Oxford University)**

In keeping with the title of the conference, the first half of my talk will review the celebrated results of Borel, Minkowski, Quillen, and others on the rational K-theory of the integers, the stable cohomology of the general linear group, and the computation of the regulator in terms of odd zeta values. The second half of the talk will cover very recent results due to many authors who have completely transformed our understanding of this field. If time permits, I plan to discuss some of the following topics: how the Borel regulator is related to certain motives of graphs constructed by Bloch, Esnault, and Kreimer; how to construct an algebraic incarnation of the Borel-Serre compactification of  $GL_n(\mathbb{Z})$ ; why the cohomology of  $GL_n(\mathbb{Z})$  has additional structures; and finally, why we expect iterated extensions of motives to appear in its unstable cohomology.

## Coffee break (11:00 - 11:30)

## Picard–Fuchs Equations of Dimensionally Regulated Feynman Integrals (11:30 - 12:30)

- **Presenter: VANHOVE, Pierre (Institut de Physique théorique)**

Feynman integrals are relative period integrals. In this lecture, I will discuss various algorithms for determining the D-module of differential equations satisfied by the Feynman integrals. In integer dimension, we have a (relative) period of a rational differential form. One approach is the use of the Griffiths-Dwork algorithms, which have been adapted to the case of non-isolated singularities, generically present for Feynman integrals. The analysis will be illustrated with a class of two-loop Feynman integrals. The study of the geometry and Hodge theory of the cubic hypersurfaces attached to two-loop Feynman integrals for generic physical parameters will be presented. We will demonstrate that the Hodge structure associated with planar two-loop Feynman graphs decomposes into mixed Tate pieces and the Hodge structures of families of hyperelliptic, elliptic, or rational curves, depending on the space-time dimension. In general dimensions, we have a twisted differential form. We will then explain how to extend the Griffiths-Dwork reduction, making particular use of the properties of the graph polynomials entering the definition of the integrand. We will examine the manner in which the twist is incorporated into the Picard-Fuchs operators.

## Lunch (12:30 - 14:30)

## Euler's constant and exponential motives (14:30 - 15:30)

- **Presenter: FRESÁN, Javier (Ecole Polytechnique)**

In the category of exponential motives over  $\mathbb{Q}$  there is a new extension of  $\mathbb{Q}(-1)$  by  $\mathbb{Q}(0)$  which does not come from classical motives. Its period matrix features Euler's constant, which one is tempted to think of as the regularised value of Riemann's zeta function at 1. I will discuss several results and open questions revolving around this extension, for example the role it plays as a "monodromy factor" for differential equations of E-functions. The talk is based on joint work with Peter Jossen.

## Coffee break (15:30 - 16:00)

## $K_2$ of elliptic curves over non-Abelian cubic and quartic fields (16:00 - 17:00)

- **Presenter: DE JEU, Rob (Vrije Universiteit)**

After a review of some earlier results on (mostly)  $K_2$  of curves, we give constructions of families of elliptic curves over certain cubic or quartic fields with three, respectively four, 'integral' elements in the kernel of the tame symbol on the curves. The fields are in general non-Abelian, and the elements linearly independent. For their integrality, we discuss a new criterion that does not ignore any torsion. We also verify Beilinson's conjecture numerically for some of the curves. This is joint work with François Brunault, Liu Hang, and Fernando Rodríguez Villegas.

# Tuesday, 4 June 2024

## Modular regulators and multiple modular values (10:00 - 11:00)

- **Presenter: BRUNAUT, François (École Normale Supérieure de Lyon)**

I will present newly constructed elements in the  $K_4$  group of modular curves, and explain how to compute their regulators in terms of L-functions of modular forms. One crucial tool is the theory of multiple modular values by Manin and Brown. This is joint work with Wadim Zudilin.

## Coffee break (11:00 - 11:30)

## Linear relations between 1-periods (11:30 - 12:30)

- **Presenter: SERTÖZ, Emre (Leiden University)**

I will sketch a modestly practical algorithm to compute all linear relations with algebraic coefficients between any given finite set of 1-periods. As a special case, we can algorithmically decide transcendence of 1-periods. This is based on the “qualitative description” of these relations by Huber and Wüstholz. We combine their result with the recent work on computing the endomorphism ring of abelian varieties. This is a work in progress with Jöel Ouaknine and James Worrell.

## Lunch (12:30 - 14:30)

## Heights and periods of limit mixed Hodge structures (14:30 - 15:30)

- **Presenter: DE JONG, Robin (University of Leiden)**

This is a report on joint work with Spencer Bloch and Emre Sertöz. We aim to establish connections between the arithmetic height of certain cycles found in the resolution of odd-dimensional nodal projective hypersurfaces defined over the rationals, and periods of limit mixed Hodge structures found in the smoothing of such hypersurfaces.

## Coffee break (15:30 - 16:00)

## Generalized Cross-Ratios (16:00 - 17:00)

- **Presenter: BLOCH, Spencer (University of Chicago)**

For  $\mathbb{P}^n$  smooth projective variety of dim.  $n$  over  $\mathbb{C}$  complex numbers;  $Y=Y_1-Y_2$   $\text{codim } r$  algebraic cycle,  $Z=Z_1-Z_2$   $\text{dim } r-1$  algebraic cycle,  $Y, Z$  disjoint support and homologous to 0. Biextension  $\mathbb{B} := H^{2r-1}(\mathbb{P}^n \setminus Y, Z; \mathbb{Q}(r))$  mixed  $\mathbb{Q}$ -HS with weights  $0, -1, -2$  and weight graded  $W_{-2} \mathbb{B} = \mathbb{Q}(1)$ ,  $W_{-1} \mathbb{B} = H^{2r-1}(\mathbb{P}^n, \mathbb{Q}(r))$ , and  $W_0 \mathbb{B} = \mathbb{Q}(0)$ . Degenerate case  $W_{-1} \mathbb{B} = (0)$  yields a Kummer extension  $0 \rightarrow \mathbb{Z}(1) \rightarrow \mathbb{B} \rightarrow \mathbb{Z}(0) \rightarrow 0$ . Such a Kummer extension carries a generalized cross-ratio  $\lambda(\mathbb{B}) \in \mathbb{C}^*$ . Examples and conjectures about generalized cross-ratios will be discussed.

## Reception (17:30 - 19:00)

# Wednesday, 5 June 2024

## Height pairings of cycles on modular varieties and L-functions (10:00 - 11:00)

- **Presenter: ZHANG, Wei (MIT)**

The Gross-Zagier formula relates the Neron-Tate height of Heegner divisors to the first central derivative of the L-function of elliptic curves over the rationals. This talk will focus on presenting several generalizations of this formula to higher dimensional modular varieties (often parameterizing abelian varieties with decorations), including conjectures made by Gan-Gross-Prasad, Kudla-Rapoport, and Yifeng Liu, and some recent developments towards them.

## Coffee break (11:00 - 11:30)

## Hypergeometric families and Beilinson's conjectures (11:30 - 12:30)

- **Presenter: KERR, Matt (Washington University, St. Louis)**

I will describe the construction of motivic cohomology classes on hypergeometric families of Calabi-Yau 3-folds using Hadamard convolutions. These are analogous to elements of the Mordell-Weil group for families of elliptic curves, and produce solutions to certain inhomogeneous Picard-Fuchs equations. This is part of a joint project with Vasily Golyshev in which we numerically verify Beilinson's conjectures in some new cases.

## Lunch (12:30 - 14:30)

## Stable I-surfaces of index 2 and generalized spin curves of genus 2 (14:30 - 15:30)

- **Presenter: PARDINI, Rita (Università di Pisa)**

An I-surface (also called a (1,2)-surface) is a complex projective surface with  $K^2=1$ ,  $h^2(O)=2$  and ample canonical class. Gorenstein stable I-surfaces are hypersurfaces of degree 10 in  $\mathbb{P}(1,1,2,5)$ . In order to study stable I-surfaces of index 2 we introduce generalized Gorenstein spin curves, namely pairs  $(C,L)$  where  $C$  is a Gorenstein curve with ample canonical class and  $L$  is a torsion-free rank 1 sheaf on  $C$  with  $\chi(L)=0$  admitting a generically injective map  $L \otimes L \rightarrow \omega_C$ . We obtain a complete classification of such pairs with  $C$  reduced of genus 2 and derive from it the classification of stable I-surfaces of index 2 with a reduced canonical curve.

## Coffee break (15:30 - 16:00)

## Atypical Hodge Loci (16:00 - 17:00)

- **Presenter: GRIFFITHS, Phillip (Institute for Advanced Study)**

In the recent works of a number of people there has emerged a new perspective on Hodge loci. A central result in that development appears in a paper by Baldi, Klingler, and Ullmo\*. In this talk, we will explain their result and give a proof. The essential step is to use the integrability conditions associated to a Pfaffian PDE system. \*Baldi, Gregorio; Klingler, Bruno; Ullmo, Emmanuel. On the distribution of the Hodge locus. Invent. Math. 235 (2024), no. 2, 441–487

# Thursday, 6 June 2024

## Algebraizability of vector bundles and motivic homotopy theory (09:30 - 10:30)

- **Presenter: BACHMANN, Tom (University of Mainz)**

I will outline a program envisioned by Mike Hopkins to prove that all topological vector bundles on certain varieties are algebraizable, and I will report on recent progress on implementing this program. In particular I will explain how to combine convergence theorems of Levine and Bousfield--Kan to construct an unstable Novikov spectral sequence in motivic homotopy theory.

## Coffee break (10:30 - 11:00)

## Evaluations of areal Mahler measure (11:00 - 12:00)

- **Presenter: LALÍN, Matilde (University of Montreal)**

The (logarithmic) Mahler measure of a non-zero rational function  $P$  in  $n$  variables is defined as the mean of  $\log |P|$  (with respect to the normalized arclength measure) restricted to the standard  $n$ -dimensional unit torus. It has been related to special values of L-functions via regulators. Pritsker (2008) defined the areal Mahler measure, which is obtained by replacing the normalized arclength measure on the standard  $n$ -torus by the normalized area measure on the product of  $n$  open unit disks. In this talk, we will investigate some similarities and differences between the two versions of Mahler measure. We will also discuss some evaluations of the areal Mahler measure of multivariable polynomials, which also yields special values of L-functions.

## Break (12:00 - 12:15)

## Motives, Periods and Species (12:15 - 13:15)

- **Presenter: HUBER-KLAWITTER, Annette (University of Freiburg)**

The Period Conjecture makes a qualitative prediction about all linear relations between the periods of motives. It is a theorem in the case of 1-motives, e.g. for numbers like logarithms or algebraic numbers or periods of elliptic curves over number fields. In joint work with Martin Kalck, we explain how to deduce dimension formulas via the structure theory of finite dimensional algebras over perfect fields.

## Free afternoon (13:15 - 17:00)

# Friday, 7 June 2024

## A topological formula for the central value of symplectic L-functions and Reidemeister torsion (10:00 - 11:00)

- **Presenter: ABDURRAHMAN, Amina (IHES)**

We give a global cohomological formula for the central value of the L-function of a symplectic representation on a curve up to squares. It involves a map  $C$ . Soule defined in his work on the Lichtenbaum conjectures. The proof relies crucially on a similar formula for the Reidemeister torsion of 3-manifolds. We sketch both analogous arithmetic and topological pictures. This is based on joint work with A. Venkatesh.

## Coffee break (11:00 - 11:30)

## Continuity principle in ramification theory (11:30 - 12:30)

- **Presenter: ABE, Tomoyuki (Kavli IPMU)**

One of the goals of ramification theory is to compute the Euler-Poincaré characteristic of a given sheaf by using an invariant measuring the ramification. Bloch's revolutionary approach to this problem is to measure using  $CH_0$ . After a long sought, this element of  $CH_0$  had finally been constructed by T. Saito, by constructing the characteristic cycle. In the first half of the talk, I'll explain an alternative construction using "continuity principle". In the second half, using the continuity principle in another way, I'll give a proof to a conjecture of Serre on the construction of Artin representation in the equal characteristic case.

## Lunch (12:30 - 14:30)

## Unlikely Intersections and applications to Diophantine Geometry (14:30 - 15:30)

- **Presenter: CAPUANO, Laura (Università di Roma III)**

The Zilber-Pink conjectures on unlikely intersections deal with intersections of subvarieties of a (semi)abelian variety or, more in general, of a Shimura variety, with "special" subvarieties of the ambient space. These conjectures generalize many classical results such as Faltings' Theorem (Mordell Conjecture), Raynaud's Theorem (Manin-Mumford Conjecture) and André-Oort Conjecture and have been studied by several authors in the last two decades. Most proofs of results in this area follow the well-established Pila-Zannier strategy, first introduced by the two authors in 2008 to give an alternative proof of Raynaud's theorem as a combination of results coming from o-minimality (Pila-Wilkie's theorem) with other Diophantine ingredients. The talk will focus on a general introduction to these problems, on some results for semi-abelian varieties and families of abelian varieties, and on applications to other problems of Diophantine nature.

## Coffee break (15:30 - 16:00)

## Sato-Tate groups of Fermat Jacobians (16:00 - 17:00)

- **Presenter: LOMBARDO, Davide (Università di Pisa)**

The arithmetic of abelian varieties is often studied through the lens of their Galois representations. Given an abelian variety  $A$  over a number field  $K$ , an important invariant is the so-called Sato-Tate group  $\operatorname{ST}(A)$ , a compact Lie group which conjecturally describes the asymptotic distribution of the characteristic polynomials of Frobenius acting on the Tate modules of  $A/K$ . The group of connected components of  $\operatorname{ST}(A)$  has particular arithmetic significance, and there exists a unique minimal extension  $L/K$  such that  $\operatorname{ST}(A_L)$  is connected. There is currently no general technique to determine  $\operatorname{ST}(A)$ , nor the extension  $L/K$ . In this talk I will describe how to compute these two invariants for the Jacobian of the curve  $y^2 = x^m + 1$  by relating them to the cohomology of (several) Fermat hypersurfaces  $X_m^n : Y_0^m + \dots + Y_{n+1}^m = 0$ . The structure of this cohomology has been studied extensively by Deligne; our application, however, will require a more detailed analysis of the action of the absolute Galois group of  $\mathbb{Q}$  on the étale cohomology of  $X_m^n$ .



# Monday, 10 June 2024

## **Primes, knots and the scaling site - Aula magna di Scienze area Pontecorvo (10:00 - 11:00)**

- **Presenter:** CONSANI, Caterina (Johns Hopkins University)

The scaling site (a Grothendieck topos) and its periodic orbits of length  $\log p$  provide a geometric construction where to interpret the well-known analogy between primes and knots. The role of the maximal abelian cover of the scaling site is played by the adèle class space of the rationals. The inverse image of a  $p$ -periodic orbit is canonically isomorphic to the mapping torus of the multiplication by the  $p$ -Frobenius in the abelianized étale fundamental group of the spectrum of the integers localized at  $p$ , thus exhibiting the linking of  $p$  with the other rational primes.

## **Coffee break - Aula magna di Scienze area Pontecorvo (11:00 - 11:30)**

## **Hyperelliptic Curves mapping to Abelian Surfaces and applications to Beilinson's Conjecture for 0-cycle - Aula magna di Scienze area Pontecorvo (11:30 - 12:30)**

- **Presenter:** GAZAKI, Evangelia (University of Virginia)

For a smooth projective variety  $X$  over an algebraically closed field  $k$  the Chow group of 0-cycles exhibits many similarities with the Picard group of a smooth projective curve. The striking difference is that in higher dimensions the Abel-Jacobi map can be very far from isomorphism. In fact, when  $k$  is the field of complex numbers and the variety  $X$  has positive geometric genus, the kernel of the Abel-Jacobi map is known to be enormous. On the other extreme, when  $k$  is the algebraic closure of the rational numbers a famous conjecture of Beilinson predicts that the Abel-Jacobi is an isomorphism. Apart from the case of curves, there is little to no evidence for this conjecture. In this talk I will present joint work with Jonathan Love where for an abelian surface  $A$  we describe a very rich collection of rational equivalences arising from hyperelliptic curves mapping to  $A$ . Additionally, we show that at least in the case when  $A$  is isogenous to a product of two elliptic curves, such hyperelliptic curves are plentiful. Namely, we give a construction that produces for infinitely many values of  $g$ , infinitely many hyperelliptic curves of genus  $g$  mapping birationally to  $A$ .

## **Lunch (12:30 - 14:30)**

## **The spectrum of Artin motives - Aula magna di Scienze area Pontecorvo (14:30 - 15:30)**

- **Presenter:** GALLAUER, Martin (University of Warwick)

I will present a complete classification of thick tensor ideals in the derived category of Artin motives (in the sense of Voevodsky) over arbitrary fields. This is joint work with Paul Balmer.

## **Coffee break - Aula magna di Scienze area Pontecorvo (15:30 - 16:00)**

## **On the second Iwasawa cohomology and rational points on elliptic curves - Aula magna di Scienze area Pontecorvo (16:00 - 17:00)**

- **Presenter:** RAMDORAI, Sujatha (University of British Columbia)

# Tuesday, 11 June 2024

## Brauer and Neron-Severi groups of surfaces over finite fields - Aula magna di Scienze area Pontecorvo (10:00 - 11:00)

- **Presenter:** GEISSER, Thomas (Rikkyo University)

For a smooth and proper surface over a finite field, the formula of Artin and Tate relates the behavior of the zeta-function at  $s=1$  to other invariants of the surface. We give a refinement which equates invariants only depending on the Brauer group to invariants only depending on the Neron-Severi group. We also give estimates of the terms appearing in the formula. This implies, for example, the largest Brauer group of an abelian surface over the field of order  $q=p^{2r}$  has order  $16q$ , and the largest Brauer group of a supersingular abelian surface over a prime field is 36.

## Coffee break - Aula magna di Scienze area Pontecorvo (11:00 - 11:30)

## Nori connectivity for squared hypersurfaces and classicality of the motivic Galois group (11:30 - 12:30)

- **Presenter:** AYOUB, Joseph (University of Zurich)

Let  $X$  be a smooth and projective variety of dimension  $n+1$  and let  $\mathcal{Y} \rightarrow S$  be the universal family of smooth hypersurfaces in  $X$  of a fixed degree. Assuming that the degree is sufficiently large, Nori proved that the cohomology of the base change  $\mathcal{Y} \times_S T$  of  $\mathcal{Y}$  along a smooth morphism  $T \rightarrow S$  coincides with the cohomology of  $X \times T$  up to degree  $2n-1$ . In particular, this gives a simple way to compute the cohomology of the local system  $(R^i f_* \mathbb{Q})_{\text{prim}}$  up to degree  $n-1$ , and after base change by any smooth morphism. We propose a version of Nori's theorem for the self product  $\mathcal{Y} \times_S \mathcal{Y}$  of the universal family. This will yield information on the higher endomorphisms of the local system  $(R^i f_* \mathbb{Q})_{\text{prim}}$ . We hope to use these endomorphisms to rule out the existence of positive degree operations on rational Betti cohomology and hence proving that the motivic Galois group is classical.

## Lunch (12:30 - 14:30)

## Explicit constructions of motivic Galois Groups - Aula magna di Scienze area Pontecorvo (14:30 - 15:30)

- **Presenter:** GONCHAROV, Alexander (Yale University)

This talk is based on the joint work with Guangyu Zhu. The category of  $\mathbb{Q}$ -mixed Hodge-Tate structures is canonically equivalent to the category of graded comodules over a graded commutative Hopf algebra  $H$  over  $\mathbb{Q}$ . The  $H$  is isomorphic to the tensor algebra of the direct sum over  $n > 0$  of  $\mathbb{C} \otimes \mathbb{Q}(n)$ , placed in the degree  $n$ , with the shuffle product. However this isomorphism is not natural, and does not work in families. We give a natural explicit construction of the Hopf algebra  $H$ . Generalizing this, we define a Hopf dg-algebra describing a dg-model of the derived category of variations of Hodge-Tate structures on a complex manifold  $X$ . Its cobar complex is a dg-model for the rational Deligne cohomology of  $X$ . The main application is explicit construction of regulators. We define refined periods. They are single-valued, and take values in the tensor product of  $\mathbb{C}^*$  and  $n-1$  copies of  $\mathbb{C}$ . We also consider a  $p$ -adic variant of the construction.

## Coffee break - Aula magna di Scienze area Pontecorvo (15:30 - 16:00)

## Enriched Hodge structures and cycles on analytic thickenings - Aula magna di Scienze area Pontecorvo (16:00 - 17:00)

- **Presenter:** SRINIVAS, Vasudevan (University at Buffalo)

This talk is a report on an ongoing project with Madhav Nori and Deepam Patel. We consider triples  $(X, A, B)$  where  $X$  is a complex analytic space,  $A, B$  are closed analytic subspaces such that  $A$  is a proper algebraic variety, and  $X \setminus B$  is a complex manifold, and  $A \setminus B$  is a submanifold. We view this as defining a representative of a germ of an analytic neighbourhood of  $A$  (the "thickening" of  $A$ ). If  $i : A \rightarrow X$  and  $j : X \setminus B \rightarrow X$  are the inclusions, we may consider cohomology groups  $H^m(A, i^*(-1)Rj_* \mathbb{Z})$  (and Tate twists). Our goal is to define a variant of Deligne-Beilinson cohomology for such objects, using Enriched Hodge structures (Bloch-Srinivas), which are "enhanced" versions of Mixed Hodge structures. We expect that our "Enriched D-B Cohomologies" would be the targets of regulators defined on suitable  $K$ -groups associated to such germs, and these would detect interesting elements in the  $K$ -theory of the germs. An example is when  $X$  is a small ball around  $A = \{0\}$  in  $\mathbb{C}^n$ , and  $B = \emptyset$ , which corresponds to the  $K$ -groups of the ring of convergent power series in  $n$  complex variables; here the underlying MHS has no information, while the "enriched" version has content. In this talk, we will indicate how the EHS's are constructed, what the corresponding Enriched DB-cohomology looks like, and discuss some simple examples.

# Wednesday, 12 June 2024

## **Framed height pairing - Aula magna di Scienze area Pontecorvo (10:00 - 11:00)**

- **Presenter: GOSWAMI, Souvik (University of Barcelona)**

In abstract Hodge theory, Deligne's splitting measures how far a mixed Hodge structure is from being real split. An allied notion, developed by S. Bloch, R.Hain et al. , is that of a height for a special class of mixed Hodge structures called Biextensions. The notion is Biextension is closely related to algebraic cycles homologous to zero. Given two such cycles in complimentary codimensions in an ambient smooth and projective variety, a certain cohomology group associated to the pair gives an example of a Biextension mixed Hodge structure. The height associated to such a Biextension is exactly same as the archimedean component of the height pairing of the two cycles developed by Bloch and Beilinson. In a previous paper (<https://doi.org/10.1112/plms.12443>), along with J.I.Burgos Gil and G. Pearlstein, the speaker defined the notion of height for oriented mixed Hodge structures. These mixed Hodge structures arises out of Bloch's higher cycles in complimentary codimensions, and is more complicated than the Biextensions.

## **Coffee break - Aula magna di Scienze area Pontecorvo (11:00 - 11:30)**

## **Relative rigid cohomology via motivic homotopy theory - Aula magna di Scienze area Pontecorvo (11:30 - 12:30)**

- **Presenter: VEZZANI, Alberto (Università Statale di Milano)**

We show how the language of motivic non-archimedean homotopy theory can be used to define p-adic cohomology theories and prove new results about them. For example, we show how to define solid relative rigid cohomology and prove a version of Berthelot's conjecture for it (joint work with V. Ertl), and how to construct Hyodo-Kato cohomology, together with its associated Clemens-Schmid chain complex, bypassing log-geometry (joint work with F. Binda and M. Gallauer).

## **Lunch (12:30 - 14:30)**

## **Regulators and derivatives of Vologodsky integrals with respect to $\log(p)$ - Aula magna di Scienze area Pontecorvo (14:30 - 15:30)**

- **Presenter: BESSER, Amnon (Ben Gurion University)**

Often, p-adic regulators for varieties with good reduction can be computed in terms of Coleman integrals. In the bad reduction case one gets meaningful results by replacing Coleman integrals with Vologodsky integrals. These integrals depend on a choice of the branch of the p-adic logarithm, determined by a choice of  $\log(p)$ . In this talk, partially based on joint work with J. Mueller and P. Srinivasan, I will explain why the derivative of the regulator with respect to the branch parameter  $\log(p)$  is an interesting "discrete regulator" and how it may be computed in terms of derivatives of Vologodsky functions with respect to  $\log(p)$ . Several examples, some of which are conjectural, will be discussed: p-adic heights, non-abelian p-adic Albanese maps, and syntomic regulators.

## **Coffee break - Aula magna di Scienze area Pontecorvo (15:30 - 16:00)**

## **Depth reduction for multiple polylogarithms - Aula magna di Scienze area Pontecorvo (16:00 - 17:00)**

- **Presenter: GANGL, Herbert (Durham University)**

The special values of the Dedekind zeta function of a number field  $F$  at integer argument  $n$  should, according to Zagier's Polylogarithm Conjecture, be expressed via a determinant of  $F$ -values of the  $n$ -th polylogarithm function. Goncharov laid out a vast program incorporating this conjecture using properties of multiple polylogarithms and the structure of a motivic Lie coalgebra. This led him to formulate his Depth Conjecture and a new strategy for solving the one given by Zagier. We report on progress in this direction since Regulators IV, some in joint work with, or developed by, S.Charlton, D.Radchenko as well as D.Rudenko and his collaborators.

# Thursday, 13 June 2024

## **On relative singular support in mixed characteristic - Aula magna di Scienze area Pontecorvo (10:00 - 11:00)**

- **Presenter:** SAITO, Takeshi (University of Tokyo)

The notion of micro support in mixed characteristic is formulated using the Frobenius--Witt cotangent bundle but the existence of the singular support is not yet known. We introduce a relative notion of micro support over a fixed regular scheme  $S$  and prove the existence of a saturation of the relative singular support on smooth schemes over  $S$ . The proof is a variation of that by Beilinson using the Radon transform.

## **Coffee break - Aula magna di Scienze area Pontecorvo (11:00 - 11:30)**

## **A quadratically enriched logarithmic zeta function - Aula magna di Scienze area Pontecorvo (11:30 - 12:30)**

- **Presenter:** WICKELGREN, Kirsten (Duke University)

André Weil proposed a beautiful connection between algebraic topology and the number of solutions to equations over finite fields in a celebrated paper from 1948: the zeta function of a variety over a finite field is simultaneously a generating function for the number of solutions to its defining equations and a product of characteristic polynomials of endomorphisms of cohomology groups. The ranks of these cohomology groups are the Betti numbers of the associated complex manifold. We enrich the logarithmic derivative of the zeta function to a power series with coefficients in the Grothendieck--Witt group of stable isomorphism classes of unimodular modular forms, using traces of powers of Frobenius in  $A_1$ -homotopy theory. We show the quadratically enriched logarithmic zeta function to be connected to the Betti numbers of the associated real manifold under certain hypotheses. This is joint work with Margaret Bilu, Wei Ho, Padma Srinivasan, and Isabel Vogt and joint work in progress with Tom Bachmann.

## **Bonus talk: The affine Grassmannian as a presheaf quotient (15:00 - 16:00)**

- **Presenter:** CESNAVICIUS, Kestutis (Orsay)

The affine Grassmannian of a reductive group  $G$  is usually defined as the étale sheafification of the quotient of the loop group  $LG$  by the positive loop subgroup. I will discuss various triviality results for  $G$ -torsors which imply that this sheafification is often not necessary.