INdAM Workshop:
Analysis and Geometry of Random Fields

BOOK OF ABSTRACTS

Wednesday, 4 September 2024 - Friday, 6 September 2024

Contents

Roots of random trigonometric polynomials with general dependent coefficients

Jürgen Angst
Université de Rennes

Abstract

We consider random trigonometric polynomials with general dependent coefficients. We show that under mild hypotheses on the structure of dependence of the latter, the asymptotics as the degree goes to infinity of the expected number of reals zeros coincides with the one in the case of independent coefficients. To the best of our knowledge, this universality statement is the first obtained in a non-Gaussian dependent context. Joint work with Oanh Nguyen and Guillaume Poly.

Correlation Structure and Resonant Pairs for Arithmetic Random Waves

Valentina Cammarota
Sapienza Università di Roma

Abstract

The geometry of Arithmetic Random Waves has been extensively investigated in the last fifteen years. In this paper we study the correlation structure among different functionals such as nodal length, boundary length of excursion sets, and the number of intersection of nodal sets with deterministic curves in different classes; the amount of correlation depends in a subtle fashion from the values of the thresholds considered and the symmetry properties of the deterministic curves. In particular, we prove the existence of resonant pairs of threshold values where the asymptotic correlation is full, that is, at such values one functional can be perfectly predicted from the other in the high energy limit. We focus mainly on the 2-dimensional case but we discuss some specific extensions to dimension 3. Joint work with Riccardo-W. Maffucci, Domenico Marinucci, Maurizia Rossi.

Multi-Scale CUSUM Tests for Time Dependent Spherical Random Fields

Alessia Caponera
Luiss University

Abstract

In this talk we investigate the asymptotic behavior of structural break tests in the harmonic domain for time dependent spherical random fields. In particular, we prove a functional central limit theorem result for the fluctuations over time of the sample spherical harmonic coefficients, under the null of isotropy and stationarity; furthermore, we prove consistency of the corresponding CUSUM test, under a broad range of alternatives, including deterministic trend, abrupt change, and a nontrivial power alternative. Our results are then applied to NCEP data on global temperature: our estimates suggest that Climate Change does not simply affect global average temperatures, but also the nature of spatial fluctuations at different scales.
Based on a joint work with Anna Vidotto and Domenico Marinucci.

TBA

Javier Carrón Duque
Istituto de Física Teórica

Abstract

TBA

Aliasing effects for random fields over the sphere

Claudio Durastanti
Sapienza Università di Roma

Abstract

This talk explores aliasing effects that arise during the reconstruction from discrete samples of spin spherical random fields defined on the sphere, with a brief discussion of the scalar case. With a fixed sampling method for obtaining field samples, we iden-

tify the location and intensity of aliases in the frequency domain for the harmonic coefficients in the Fourier decomposition of the random field and assess the impact of aliasing errors on the angular power spectrum. Finally, we show that band-limited spin random fields can be free from aliasing if a sufficiently large number of samples is taken.

The variance linearity for the nodal volume of Gaussian fields

Louis Gass
Université du Luxembourg

Abstract

In this talk, I will present a short proof of the linearity of the variance for the nodal volume, or the number of critical points, of a Gaussian field, under very mild conditions on the covariance function. Notably, this proof bypasses the usual isotropy condition, as well as the intricate computation of the second chaos component of the absolute value of the Jacobian determinant. The proof is general in nature and can be extended to a broader class of local functionals of Gaussian fields.

L^∞ norm of generic chaotic eigenfunctions

Maxime Ingremeau
Université Côte d'Azur

Abstract

A fundamental question in quantum chaos is to understand how localised/delocalised eigenfunctions of the Laplacian can be, in particular on manifolds of negative curvature. One way of addressing the question of delocalisation is to understand how large the L^∞ norms of the eigenfunctions can be. While there are several conjectures on the behaviour of these L^∞ norms, very little has been proved. In this talk, we will show adding small random perturbations to the Laplacian can help to prove L^∞ bounds on the eigenfunctions. This is joint work with Martin Vogel.

Limit theorems and geometric functionals for spatiotemporal

random fields

Nikolai Leonenko
Cardiff University

Abstract

The talk will be devoted to three interlinked directions of research:

1. High-level moving excursions for spatiotemporal random fields (joint work with M.D.Ruiz-Medina) We analyse the limiting distribution of sojourn measures above moving levels of spatiotemporal Gaussian random fields with long range dependence (LRD) under increasing domain asymptotics in space and/or in time [1,2].

2. Limit theorems for the volumes of excursion sets of heavy-tailed random fields with LRD (joint results with V.Anh, A.Olenko and V. Vaskovich) [3.4] We analyse the limiting distribution of sojourn measures for Student and Fisher-Snedecor random fields with LRD.

3. Four moments theorems for weighted functionals of non-linear transformation of stationary processes with cyclical LRD (joint results with A.V.Ivanov, M.D.Ruiz-Medina and I. Savich)[5]

Using four moments theorems the limiting Gaussian distribution of weighted functionals of non-linear transformations of Gaussian stationary processes, having multiple singular spectra, is derived. This result is obtained under very general conditions on the weight function. Specifically, it is assumed that the modulus-square of the Fourier transform of the weight function weakly converges to an atomic measure, whose atoms do not coincide with the spectral singularities of the stationary process considered. A multivariate version of these results is studied.
Some other problems related to four moments theorems for non-Gaussian settings are discussed [6,7].

References

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[4] Anh, V.V., Leonenko, N.N., Olenko, A and Vaskovich, V (2019) On rate of convergence in non-central limit theorems, Bernoulli, 25, N4A, 2920-2948

[5] Ivanov A.V., Leonenko N.N, Ruiz-Medina, M.D. and Savich, I.N. (2013) Limit theorems for weighted non-linear transformations of Gaussian processes with singular spectra, Annals of Probability, vol. 41, No 2, 1088-1114

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[7] Bourguin, S.; Campese, S., Leonenko, N. and Taqqu, M. S.(2019) Four moments theorems on Markov chaos, Annals of Probability , vol. 47 , no. 3, 1417—1446.

Limit theorems for p-domain functionals of stationary Gaussian fields

Leonardo Maini
Università di Milano-Bicocca

Abstract

We investigate central and non-central limit theorems for integral functionals of subordinated Gaussian fields on the Euclidean space, as the integration domain grows. In particular, we consider the case of p-domain functionals, where the domain can be written as the Cartesian product of p domains that (possibly) grow at different rates. First, we assume that the covariance function of the Gaussian field is separable and thoroughly investigate under which conditions the study of p-domain functionals can be reduced to that of some simpler and classical one-domain functionals. When the considered functionals are in a fixed Wiener chaos, we also provide a quantitative version of the previous result, which improves some bounds in the literature. Second, we extend our study beyond the separable case, by investigating what can be inferred when the covariance function is either in the Gneiting class or is additively separable. Based on a joint work with Nikolai Leonenko, Ivan Nourdin and Francesca Pistolato.

Spectral complexity of deep neural networks

Domenico Marinucci
Università di Roma Tor Vergata

Abstract

It is well-known that randomly initialized, push-forward, fully-connected neural networks weakly converge to isotropic Gaussian processes, in the limit where the width of all layers goes to infinity. In this paper, we propose to use the angular power spectrum of the limiting fields to characterize the complexity of the network architecture. In particular, we define sequences of random variables associated with the angular power spectrum, and provide a full characterization of the network complexity in terms of the asymptotic distribution of these sequences as the depth diverges. On this basis, we classify neural networks as low-disorder, sparse, or high-disorder; we show how this classification highlights a number of distinct features for standard activation functions, and in particular, sparsity properties of ReLU networks. Our theoretical results are also validated by numerical simulations.
Joint work with Simmaco Di Lillo, Michele Salvi e Stefano Vigogna.

TBA

Ivan Nourdin
Université du Luxembourg

Abstract

TBA

On the tightness of nodal volumes

Giovanni Peccati
Université du Luxembourg

Abstract

Let f be a (possibly multidimensional) stationary Gaussian field on a Euclidean space. Given a bounded set A, we define $\nu(A)$ to be the volume of the restriction

to A of the (random) nodal set of f. In this talk, I will explore the problem of characterizing the tightness of the (suitably rescaled) random measure ν in the classical large-domain limit - where ν typically exhibits Gaussian fluctuations. I will mostly discuss two situations: (i) smooth stationary fields with square-integrable covariances, in which case tightness can be proved by a combination of Kac-Rice formulae (via the recent Gass/Stecconi approach) and a criterion by Bickel and Wichura (1973), and (ii) Berry's planar random waves, for which the quest for tightness is still open and some intriguing connections with hyperuniform point processes can be established. Part (i) is based on a work in progress with L. Gass, whereas Part (ii) draws from a 2023 paper written in collaboration with M. Notarnicola and A. Vidotto.

Lipschitz-Killing curvatures for excursion sets of spin random fields on SO(3)

Francesca Pistolato
Université du Luxembourg

Abstract

In the present era, there is a growing interest in modeling data not only with scalar values but also with more sophisticated algebraic structures. An important example are spin spherical random fields, that can be defined as random sections of a bundle of the 2-sphere. These fields manifest in both gravitational lensing data and Cosmic Microwave Background polarization data, which can be seen as vectors on the complex plane. Motivated by studying anisotropies and divergence from Gaussianity of the latter, we study the excursion sets of their real part by means of their Lipschitz-Killing curvatures, that are geometric functionals such as, in dimension 3, the volume, the surface area, the cross-sectional diameter and the Euler characteristic. Without requiring the isotropy of the field, it is possible to compute explicitly the expectation of such functionals in terms of the spin parameter and the level of the excursion. The talk is based on the joint work "Expected Lipschitz-Killing curvatures for spin random fields and other non-isotropic fields" (2024) with M. Stecconi (submitted, arXiv:2406.04850).

Random homogeneous sums and applications to universality

Guillaume Poly

Université de Rennes

Abstract

in this talk we will review some properties of homogeneous sums and describe how to use them to obtain universal limit theorems for geometric functionals of level sets. If time permits, some conjectures regarding the complete characterization of all possible limits in distribution of such objects will be discussed.

Quadratic variations and angular power spectrum estimation

Radomyra Shevchenko
Université Côte d'Azur

Abstract

We use the simple quadratic variations ansatz to construct some estimators for the angular power spectrum C_ℓ of an isotropic Gaussian random field on the unit sphere. We consider the discrete infill asymptotics along a single geodesic line which allows us to deal with missing observations. In this setup, we study the limiting behaviour of some quadratic functionals, derive the asymptotic properties of the estimators, and discuss some further applications.

Phase Singularities of Independent Berry Random Waves with Distinct Energy Levels

Krzysztof Smutek
Université du Luxembourg

Abstract

Consider a sequence of complex monochromatic Gaussian random waves with an associated sequence of energy levels that diverges to infinity. The asymptotic fluctuation in the number of corresponding zeros in a well-behaved domain has been characterized by Nourdin, Peccati, and Rossi (2019), with subsequent refinements by Peccati and Vidotto (2020) and by Notarnicola, Peccati, and Vidotto (2023). An analogous problem has also been solved for Arithmetic Random Waves by Dalmao, Nourdin, Peccati, and Rossi (2019). In this talk, we will show how a simple devi-

ation from monochromaticity—choosing independent sequences of energies for the real and imaginary parts of the wave—perturbs such a nodal statistic and generates new non-trivial phenomena, related in particular to variance asymptotics and full correlation results.

Brownian Submanifolds

Michele Stecconi
Université du Luxembourg

Abstract

We consider a Brownian motion in spaces of smooth functions from R^d to R^r (or, more generally, a suitably regular stochastic process of C^k sections of a vector bundle on a manifold) and consider the evolution in time of the nodal set $t \mapsto Z_t$, as a random process of subsets. By standard considerations, with probability one, Z_t is a smooth submanifold at almost every time t, so the question is what happens when it is not? What is the smallest class of subsets in which we can fit this process? We show that, under natural regularity and non-degeneracy assumptions, with probability one, for all times t, the nodal set Z_t is diffeomorphic to the level set of a Morse function. This means that if Z_t is not a submanifold, then it just has one singular point of the simplest kind and at that moment the evolution of the geometry is that of a "surgery", the simplest example of a cobordism, and it can be fully understood by means of a generalized version of Morse's lemma. We also show that the set of times where this happens has exactly Hausdorff dimension $= \frac{1}{2}$ and that it is isometric to the set of zeroes of a standard real Brownian motion. As a first corollary, we derive the Holder regularity of the real valued process defined as the nodal volume of Z_t. (Based on a work with Pierre Perruchaud)

The Geometry of Time-Dependent Spherical Random Fields

Anna Vidotto
Università di Napoli Federico II

Abstract

In this talk, we consider fluctuations over time for the area of the excursion sets and the length of level curves of time-dependent Gaussian spherical random fields. We

focus on both long and short memory assumptions; in the former case, we show that the fluctuations are dominated by a single Wiener chaotic component and the existence of cancellation levels where the variance is asymptotically of smaller order. In the short memory case, we show that all Wiener chaoses contribute in the limit, no cancellation occurs and a Central Limit Theorem can be established by Breuer-Major type arguments. The talk is based on two articles written together with Domenico Marinucci and Maurizia Rossi.