On the tightness of nodal volumes

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Abstract

Let f be a (possibly multidimensional) stationary Gaussian field on a Euclidean space. Given a bounded set A, we define $\nu(A)$ to be the volume of the restriction to A of the (random) nodal set of f. In this talk, I will explore the problem of characterizing the tightness of the (suitably rescaled) random measure ν in the classical large-domain limit - where ν typically exhibits Gaussian fluctuations. I will mostly discuss two situations: (i) smooth stationary fields with square-integrable covariances, in which case tightness can be proved by a combination of Kac-Rice formulae (via the recent Gass/Stecconi approach) and a criterion by Bickel and Wichura (1973), and (ii) Berry's planar random waves, for which the quest for tightness is still open and some intriguing connections with hyperuniform point processes can be established. Part (i) is based on a work in progress with L. Gass, whereas Part (ii) draws from a 2023 paper written in collaboration with M. Notarnicola and A. Vidotto.