

Geometric integration meets data-driven dynamical systems

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Koopman operators globally linearise nonlinear dynamical systems, and their spectral information can be a powerful tool for analysing and decomposing such systems. There has been a recent flurry of activity in data-driven computations of the spectral properties of Koopman operators. However, Koopman operators are infinite-dimensional, making the computation of their spectral information a considerable challenge. In this talk, we will combine ideas from geometric integration with dynamic mode decomposition (DMD), one of the most popular algorithms for analysing Koopman operators. We introduce `measure-preserving extended dynamic mode decomposition` (`mpEDMD`), the first Galerkin method whose eigendecomposition converges to the spectral quantities of Koopman operators for general measure-preserving dynamical systems. `mpEDMD` is a data-driven algorithm based on an orthogonal Procrustes problem that enforces measure-preserving truncations of Koopman operators using a general dictionary of observables. It is flexible and easy to use with any pre-existing DMD-type method and with different data types. Enforcing this structure is crucial to its convergence and qualitative behaviour. We prove the convergence of `mpEDMD` for projection-valued and scalar-valued spectral measures, spectra, and Koopman mode decompositions. For the case of delay embedding (Krylov subspaces), our results include the first convergence rates of the approximation of spectral measures as the size of the dictionary increases. We demonstrate `mpEDMD` on a range of challenging examples, its increased robustness to noise compared with other DMD-type methods, and its ability to capture the energy conservation and cascade of a turbulent boundary layer flow with a Reynolds number $> 6 \times 10^4$ and a state-space dimension $> 10^5$.

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