

# Detecting the numerical of ill posedness in delay differential equations

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Given a set of matrices  $A_i \in \mathbb{C}^{n \times n}$  and a set of analytic functions  $f_i : \mathbb{C} \mapsto \mathbb{C}$ , we consider a regular matrix-valued function  $\mathcal{F}(\lambda) = \sum_{i=0}^d f_i(\lambda) A_i$ , that is  $\det(\mathcal{F}(\lambda))$  is not identically zero for  $\lambda \in \mathbb{C}$ . An interesting problem consists in the computation of the nearest singular function  $\tilde{\mathcal{F}}(\lambda) = \sum_{i=0}^d f_i(\lambda) (A_i + \Delta A_i)$ , with respect to the Frobenius norm. For example, this problem has particular importance in the context of delay differential algebraic equations, where a function in the form  $\mathcal{D}(\lambda) = \lambda E - A - B e^{-\tau \lambda}$  is studied. Indeed in this setting and in presence of small delays  $\tau$ , the ill posedness of the problem may be connected with the numerical singularity of the function  $\mathcal{D}(\lambda)$ , even if the pencil  $\lambda E - A$  is regular. We will provide a general overview of the problem, describing the possible issues connected with the lack of robustness of the differential equation, associated with destabilizing perturbations of  $\mathcal{D}(\lambda)$ . Moreover we propose a method for the numerical approximation of the function  $\mathcal{F}(\lambda)$ , which rephrases the matrix nearness problem for the matrix-valued function into an equivalent optimization problem. Nevertheless this problem turns out to be highly non-convex. To solve it, we propose a two level procedure, which introduces a constrained gradient system of differential equations in the inner iteration and a Newton-like method for the optimization of the perturbation size in the outer one. This is a joint work with Nicola Guglielmi (GSSI).

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