

Ricercatori in Algebra e Geometria 2024

Wednesday, September 25, 2024 - Friday, September 27, 2024

Università degli Studi di Milano La Statale
Program

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Wednesday, September 25, 2024

K3 surfaces associated to varieties of generalized Kummer type and applications to the Hodge conjecture -

Aula A (1:30 PM - 2:30 PM)

- **Presenter: FLOCCARI, Salvatore (Leibniz University Hannover, Germany)**

Varieties of generalized Kummer type (Kum^n -type) are one of the two infinite series of known hyper-Kähler varieties, the other being given by deformations of Hilbert schemes of points of K3 surfaces. I will explain how any variety K of Kum^n -type has an associated K3 surface S which is geometrically related to it via a moduli of stable sheaves on S . Building upon the work of O'Grady, Markman, Voisin and Varesco, we use this construction to prove the Hodge conjecture for all powers of many K3 surfaces of Picard rank 16. We further deduce that the Hodge conjecture holds for any abelian fourfold of Weil type with discriminant 1 as well as its powers, extending a result of Markman.

Break - Aula A (2:30 PM - 3:00 PM)

An overview of isotropic motives - Aula A (3:00 PM - 4:00 PM)

- **Presenter: TANANIA, Fabio (Technische Universität Darmstadt, Germany)**

The theory of motives, envisioned by Grothendieck and further developed by Voevodsky, provides a systematic approach to the study of different cohomology theories for algebraic varieties. In this talk, I will give a brief overview of the theory of motives, with a particular focus on isotropic motives. Roughly speaking, isotropic motivic categories are obtained from their classical counterparts by “annihilating” the motives of all anisotropic varieties. The aim of the talk is to present the main features of isotropic motives, including their relation to algebraic cycles modulo numerical equivalence, and to well known algebraic objects coming from classical homotopy theory.

Coffee break - Aula A (4:00 PM - 4:30 PM)

The P=W paradigm for compact hyperkähler manifolds - Aula A (4:30 PM - 5:30 PM)

- **Presenter: MAURI, Mirko (École Polytechnique, France)**

The $P=W$ paradigm for compact hyperkähler manifolds suggests surprising relations between degenerations of these manifolds and Lagrangian fibrations on top of them. I will exemplify this principle in two instances: 1. the perverse-Hodge octahedron, i.e., a 3D enhancement of the classical Hodge diamond; 2. the ubiquity of tori as fiber of Lagrangian fibrations, vanishing cycles of type III degenerations, and now also as deeper strata of type II degenerations. This talk is based on a joint project with D. Huybrechts and an ongoing project with P. Engel.

Thursday, September 26, 2024

Building examples of hyperkähler varieties - Aula A (9:30 AM - 10:30 AM)

- **Presenter: BERTINI, Valeria (Università di Genova)**

One of the most fascinating and mysterious feature of smooth compact hyperkähler manifolds is that it is extremely difficult to produce new examples. The situation significantly improves when one moves to the singular setting, which has attracted increasing interest in the last few years. In this talk I will introduce some of the possible notions of singular hyperkähler varieties, focusing on techniques to produce new examples, as taking quotients under symplectic actions or starting from the so-called singular FK3s. The talk is based on a joint work with A. Grossi, M. Mauri, E. Mazzon and a work in progress with F. Denisi, E. Fatighenti e A. Grossi.

Coffee break - Aula A (10:30 AM - 11:00 AM)

Rational points on Fano varieties - Aula A (11:00 AM - 12:00 PM)

- **Presenter: PIEROPAN, Marta (Utrecht University, The Netherlands)**

Fano varieties form one of the fundamental classes of building blocks in the birational classification of algebraic varieties. In this talk I will discuss how their special geometric properties can be used to study their arithmetic. I will focus on a few conjectures about their rational points over number fields (potential density, Manin's conjecture) and how they can be investigated by determining the asymptotic behavior of certain counting functions of rational points. All the main objects involved will be introduced and illustrated by examples. A summary of the literature on the topic will be discussed, including my own contribution.

Lunch break - Aula A (12:00 PM - 1:45 PM)

The non-degeneracy of Enriques surfaces - Aula A (1:45 PM - 2:45 PM)

- **Presenter: ROTA, Franco (University of Glasgow, UK)**

In 1910, Fano described birational models of Enriques surfaces realized as intersections of cubics in P^5 . These Fano models are closely related with the geometry and combinatorics of elliptic fibrations on the Enriques surface. In this talk I will discuss an invariant introduced by Dolgachev and Cossec, called non-degeneracy, which measures the singularities of the Fano models of an Enriques surface. Non-degeneracy is hard to compute and its behaviour in moduli is not well understood. I will present results in these directions, which were obtained jointly with R. Moschetti and L. Schaffler.

Coffee break - Aula A (2:45 PM - 3:30 PM)

"Anti-litaka" theorem in characteristic $p > 0$ - Aula A (3:30 PM - 4:30 PM)

- **Presenter: BRIVIO, Iacopo (Harvard University, USA)**

Given a fibration of complex projective manifolds $f: X \rightarrow Y$ with general fiber F , the famous litaka conjecture predicts the inequality $\kappa(K_X) \geq \kappa(K_F) + \kappa(K_Y)$. Recently Chang has shown that, when the stable base locus of $-K_X$ is vertical over Y , a similar statement holds for the anticanonical divisor: $\kappa(-K_X) \leq \kappa(-K_F) + \kappa(-K_Y)$. Both litaka's conjecture and Chang's theorem are known to fail in positive characteristic. In this talk I will introduce a new class of "arithmetically general" positive characteristic varieties with negative canonical bundle, and show that Chang's theorem can be recovered when the general fiber F belongs to this class. Based on joint work with Marta Benozzo and Chi-Kang Chang.

Poster session - Aula A (5:00 PM - 6:30 PM)

Friday, September 27, 2024

Correspondences acting on constant cycle curves on K3 surfaces - Aula A (9:00 AM - 10:00 AM)

- **Presenter: TORELLI, Sara (Università di Roma Tre)**

Constant cycle curves on a K3 surface X are curves whose points all define the Beauville-Voisin class in the Chow group of X . They were first considered by Huybrechts and Voisin as a generalization of the notion of rational curve, which is still not fully understood. In this talk, we introduce correspondences $Z \subseteq X \times X$ over \mathbb{C} acting on constant cycle curves, and we study geometric examples that can potentially improve our current understanding. More precisely, for a general primitively polarised K3 surface (X, H) of genus $g \geq 2$, we consider for any $k \geq 2$ and any $0 \leq \delta \leq p$ the locus $Z_{\{k, \delta\}}(X, H) \subseteq X \times X$ of pairs of points (p, q) contained in some δ -nodal curve C with the property that $p - q$ is k -torsion in the Jacobian of the normalization of C . We prove that this locus is not empty of the expected dimension $2g$ if and only if a certain Brill-Noether number is non negative, and that, when not empty, it gives the desired examples. This is part of a work in progress with Andreas Knutsen.

Coffee break - Aula A (10:00 AM - 10:45 AM)

Diagrammatic Calculus for Singular Bott-Samelson Varieties - Aula A (10:45 AM - 11:45 AM)

- **Presenter: PATIMO, Leonardo (Università di Pisa)**

Singular Bott-Samelson varieties provide resolutions of singularities for Schubert varieties in partial flag varieties. The cohomology of these varieties can be treated as a bimodule, an algebraic object, which contains crucial information about the representation theory of reductive algebraic groups. Using a diagrammatic language, we study these bimodules and describe a basis, called "singular light leaves", for their space of morphisms. The construction of this basis has concrete applications in computing character formulas for reductive groups, as well as theoretical implications, paving the way for a diagrammatic definition of the singular Hecke category in representation theory. This is a joint project with B. Elias, H. Ko, and N. Libedinsky.

Automorphisms of Hyperkähler manifolds - Aula A (11:45 AM - 12:45 PM)

- **Presenter: GROSSI, Annalisa (Università di Bologna)**

Automorphisms of Hyperkähler manifolds are studied for many different reasons: among the others, to construct examples of irreducible symplectic varieties as quotients of smooth HK's by a symplectic group of automorphism, or to find examples of Enriques manifolds as étale quotients of HK manifolds. After a review of the known results about automorphisms of HK manifolds I will focus on the OG10 sporadic example and I will present a recent result concerning symplectic rigidity, and another concerning non-existence of Enriques manifolds as étale quotients of OG10. The talk is based on a joint work with L. Giovenzana, Onorati and Veniani and on a joint work in progress with Billi, L. and F. Giovenzana.