

Estimation of spectral gaps for sparse symmetric matrices

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We propose and analyze an algorithm for identifying spectral gaps of a real symmetric matrix A by simultaneously approximating the traces of spectral projectors associated with multiple different spectral slices. Our method utilizes Hutchinson's stochastic trace estimator together with the Lanczos algorithm to approximate quadratic forms involving spectral projectors. Instead of focusing on determining the gap between two particular consecutive eigenvalues of A , we aim to find all gaps that are wider than a specified threshold. By examining the problem from this perspective, and thoroughly analyzing both the Hutchinson and the Lanczos components of the algorithm, we obtain error bounds that allow us to determine the numbers of Hutchinson's sample vectors and Lanczos iterations needed to ensure the detection of all gaps above the target width with high probability. In particular, we conclude that the most efficient strategy is to always use a single random sample vector for Hutchinson's estimator and concentrate all computational effort in the Lanczos algorithm. Our numerical experiments demonstrate the efficiency and reliability of this approach.

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