

# Matrix-oriented discretisations of phase-field and high-order diffusion problems

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We consider a large class of arbitrarily high-order evolutionary PDEs, including phase-field models and polyharmonic reaction-diffusion problems on rectangular domains with Neumann boundary conditions. We propose a matrix-oriented approach that, after full discretisation, requires the solution of a sequence of algebraic matrix equations of Lancaster type. To this end, we apply lumped finite elements in space and a selection of time solvers. Hence, we solve the Allen-Cahn (2nd order), Cahn-Hilliard (4th order), polyharmonic diffusion problems ( $2m$ -th order,  $m \in \mathbb{N}$ ), and polyharmonic reaction-diffusion systems. Since these models exhibit high-rank spatial solutions, we employ the so called reduced method in the spectral space. Since the Neumann matrices arising from lumped FEM are slightly different from the corresponding well-known matrices arising from central finite differences, we derive ex-novo the needed eigendecompositions in closed form. Moreover, we provide analytical bounds for the condition number of the vectorised counterpart to show that ill-conditioning can be avoided by our matrix approach. Numerical experiments illustrate the computational performances of the matrix-oriented discretisation in terms of execution time, accurate simulation of high-rank solutions such as Turing patterns and good conditioning.

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