

# Blocking structures, approximation, and preconditioning

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Large block-structured matrices with Toeplitz-type blocks of different sizes frequently arise in various applications, but pose computational issues when solving the associated linear systems. In our setting, the matrices  $A_n$  are composed of (block rectangular) unilevel Toeplitz blocks defined by rectangular  $s \times t$  matrix-valued generating functions. Under mild assumptions on the block dimensions, the asymptotic distribution of the singular values of the associated matrix sequences is recently known. Moreover, when the singular value symbol is Hermitian, the spectral symbol coincides with the singular value symbol.

Based on these theoretical findings, we develop a method to construct simplified block matrices that approximate the original matrices. These simplified matrices offer two key advantages: (a) they maintain the same singular value distributions as  $\{A_n\}_n$ ; (b) they enable the solution of linear systems in  $O(n \log n)$  arithmetic operations.

In this way, we propose a natural preconditioning strategy for linear systems with coefficient matrix  $A_n$ . We provide detailed singular value and spectral analyses of the preconditioned matrix sequences, and validate our approach through numerical experiments concerning the convergence of various (preconditioned) Krylov solvers

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