Due Giorni di Algebra Lineare Numerica e Applicazioni

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## Iterated $\ell^2 - \ell^q$ regularization

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In numerous fields of science and engineering we are faced with problems of the form  $Ax+\eta = b^{\delta}$ , where A\in\mathbb{R}^{m\times n} is a large and severely ill-conditioned matrix, i.e., such that its singular em > noise < /em >, and {b}^\delta\in\mathbb{R}^m represents the measured and perturbed data. We will assume that the noise the em > discrete ill-posed inverse problems < /em >. Due to the ill-conditioning of A and the presence of the noise {\text{versence}}, the naive of the noise {\text{versence}} and the presence of the noise {\text{versence}}. The naive of the noise {\text{versence}} and the presence of the noise {\text{versence}} and

To reduce the sensitivity of the problem to the presence of noise, we consider the  $\langle ell^2 - \langle ell^2q$  regularization  $\arg \min_x \frac{1}{2} ||Ax - b^{\delta}||_2^2 + \frac{\mu}{q} ||Lx||_q^q$ , where  $0 < q \le 2$ ,  $\mu > 0$  is the so-called regularization parameter,  $L \langle in \rangle mathbb{R}^{f_s} (s \land imesn)$  is referred to as regularization operator, and  $\langle || \langle z \rangle || q^q = \langle un \rangle (i = 1)^{f_n} \langle lvertz \rangle_i \langle rvert^2q$ . We refer to latter quantity as  $\langle ell^2q - norm$  for any  $0 < q \langle leq2$ , even though for q < 1 it is not a norm as it does not satisfy the triangular inequality; see, e.g., [2] for a discussion on  $\langle ell^2p - \langle ell^2q$  regularization and [1] for a software implementation.

To further improve the quality of the computed solutions and to provide a theoretically sound con-

vergence analysis, we consider the refinement strategy  $\begin{cases} r^{(k)} = b^{\delta} - Ax^{(k)}, \\ h^{(k)} = \arg\min_{h} \frac{1}{2} \|Ah - r^{(k)}\|_{2}^{2} + \frac{\mu}{q} \|Lh\|_{q}^{q}, \\ x^{(k+1)} = x^{(k)} + h^{(k)}. \end{cases}$ 

We show that, under reasonable assumptions, the iterations are stopped after finitely many steps, if the Discrepancy Principle is employed as stopping criterion, and that the resulting method is a <em>regularization method</em>. Selected numerical results show the good performances of the proposed approach.<br/>

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