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Iterated $\ell^2 - \ell^q$ regularization

Monday, 20 January 2025 16:00 (2 hours)

In numerous fields of science and engineering we are faced with problems of the form $Ax+\eta = b^\delta$,*where*A\in\mathbb{R}^{m\times n}*isalargeandseverelyill−conditionedmatrix, i.e., suchthatitssingular*
em > noise < /em >, and{b}^\delta\in\mathbb{R}^m*representsthemeasuredandperturbeddata.Wewillassumethatthenoiseth*
*em > discreteill−posedinverseproblems < /em > .Duetotheill−conditioningof*A*andthepresenceofthenoise*{\eta}*, thenaive*

To reduce the sensitivity of the problem to the presence of noise, we consider the \ell^2 − \ell^q regularization $\arg\min_x \frac{1}{2}\left\|Ax - b^\delta\right\|_2^2 + \frac{\mu}{q}\left\|Lx\right\|_q^q$,*where*0<q$\leq$ 2, $\mu > 0$ is the so-called regularization parameter, L\in\mathbb{R}^{s\timesn} is referred to as regularization operator, and \|\|z\|\|q^q=\sum\{i = 1}^{n}\lvertz_i\rvert^q. We refer to latter quantity as \ell^q-norm for any $0 <$ q\leq2, even though for $q < 1$ it is not a norm as it does not satisfy the triangular inequality; see, e.g., [2] for a discussion on \ell^p − \ell^q regularization and [1] for a software implementation.

To further improve the quality of the computed solutions and to provide a theoretically sound convergence analysis, we consider the refinement strategy $\begin{cases} r^{(k)} = b^\delta - Ax^{(k)}, \\ h^{(k)} = \arg\min_h \frac{1}{2}\left\|Ah - r^{(k)}\right\|_2^2 + \frac{\mu}{q}\left\|Lh\right\|_q^q, \\ x^{(k+1)} = x^{(k)} + h^{(k)}. \end{cases}$

We show that, under reasonable assumptions, the iterations are stopped after finitely many steps, if the Discrepancy Principle is employed as stopping criterion, and that the resulting method is a regularization method. Selected numerical results show the good performances of the proposed approach.

References
   A. Buccini and L. Reichel, Software for limited memory restarted \ell^p − \ell^q minimization methods using generalized Krylov subspaces, Electronic Transactions on Numerical Analysis, 61, 66-91 (2024). A. Buccini, M. Pragliola, L. Reichel, and F. Sgallari, A comparison of parameter selection rules for \ell^p−\ell^q$*minimization, Annalidell′UniversitdiFerrara*, 68, 441−463(2022). < /li >
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