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Minimum-norm solutions of the non-symmetric semidefinite Procrustes problem

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Given two matrices $X, B \in \mathbb{R}^{n\times m}$ and a set $\mathcal{A} \subseteq \mathbb{R}^{n\times n}$, a Procrustes problem consists in finding a matrix $A \in \mathcal{A}$ such that the Frobenius norm of $AX - B$ is minimized. When $\mathcal{A}$ is the set of the matrices whose symmetric part is positive semidefinite, we obtain the so-called non-symmetric positive semidefinite Procrustes (NSPSDP) problem. The NSPSDP problem arises in the estimation of compliance or stiffness matrix in solid and elastic structures.

If X has rank r, Baghel et al. in 2022 proposed a three-step semi-analytical approach:

(1) construct a reduced NSPSDP problem in dimension $r \times r$,

(2) solve the reduced problem by means of a fast gradient method with a linear rate of convergence, and

(3) post-process the solution of the reduced problem to construct a solution of the larger original NSPSDP problem.

In this talk, we revisit this approach by Baghel et al. and we identify an unnecessary assumption used by the authors leading to cases where their algorithm cannot attain a minimum and produces solutions with unbounded norm. In fact, revising the post-processing phase of their semi-analytical approach, we show that the infimum of the NSPSDP problem is always attained, and we show how to compute a minimum-norm solution. We also prove that the symmetric part of the computed solution has minimum rank bounded by r, and that the skew-symmetric part has rank bounded by $2r$. Several numerical examples show the efficiency of this algorithm, both in terms of computational speed and of finding optimal minimum-norm solutions.

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