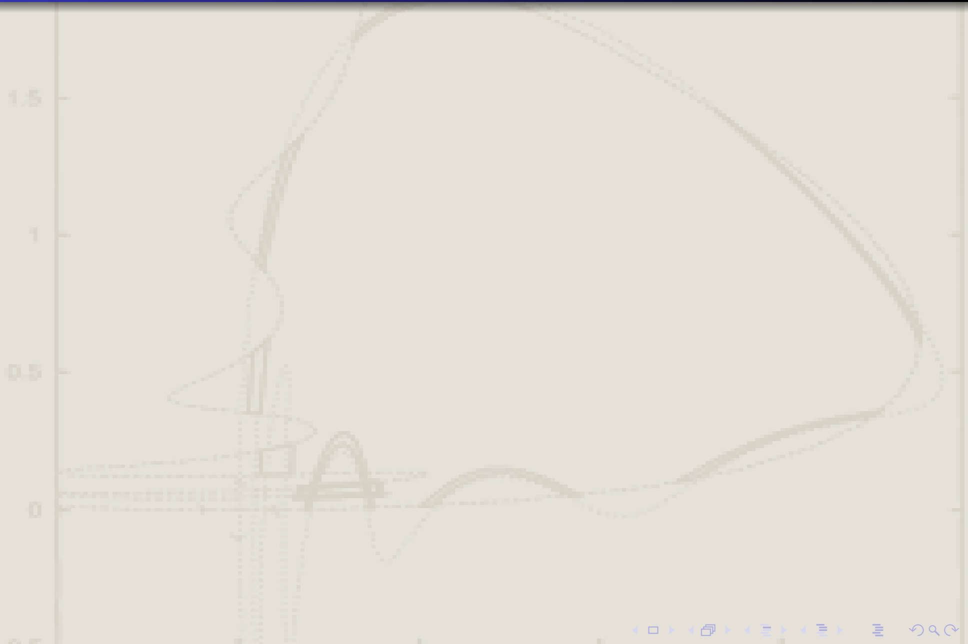
Dynamical Systems

Claudio Bonanno, Gianluigi Del Magno, Stefano Galatolo, Paolo Giulietti

What do you mean by "dynamical systems"?



What do you mean by "dynamical systems"?

From Wikipedia “Dynamical systems theory”

"*Dynamical systems theory is an* ***area of mathematics*** *used to describe the behavior of complex dynamical systems, usually by employing differential equations or difference equations. When differential equations are employed, the theory is called continuous dynamical systems. From a physical point of view, continuous dynamical systems is a generalization of classical mechanics [...]. When difference equations are employed, the theory is called discrete dynamical systems. [...].*
This theory deals with the long-term qualitative behavior of dynamical systems, and studies the nature of, and when possible the solutions of, the equations of motion of systems that are often primarily mechanical or otherwise physical in nature, such as planetary orbits and the behaviour of electronic circuits, as well as systems that arise in biology, economics, and elsewhere. Much of modern research is focused on the study of chaotic systems and bizarre systems.

What do you mean by "dynamical systems"?

Dynamical system theory as developed by:

Kolmogorov, Arnold, Moser, Sinai, Smale, Anosov, Ruelle, Bowen,...

What do you mean by "dynamical systems"?

Dynamical system theory as developed by:

Kolmogorov, Arnold, Moser, Sinai, Smale, Anosov, Ruelle, Bowen,...

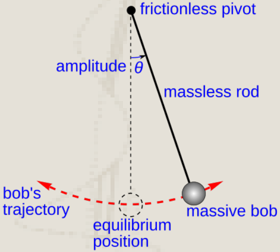
Fields medals (≥ 2000):

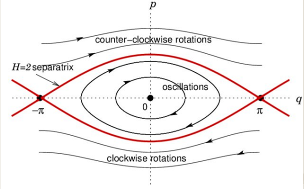
T. Tao (06), E. Lindenstrauss (10), A. Avila, M. Mirzakhani (14)

EMS prizes (≥ 2000):

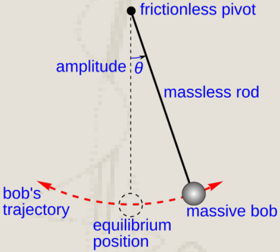
W. Tucker (04), C. Ulcigrai (12), S. Filip (20), A. Kanigowski (24)

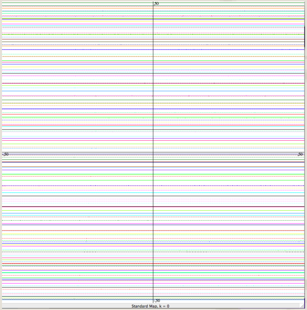
An example: perturbation of a pendulum



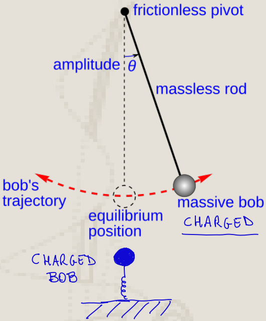


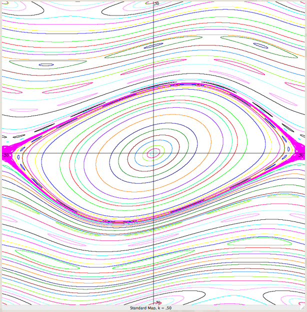
An example: perturbation of a pendulum



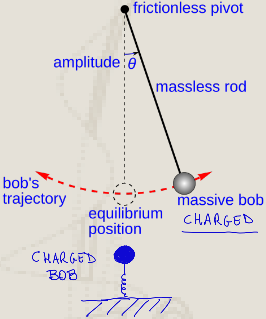


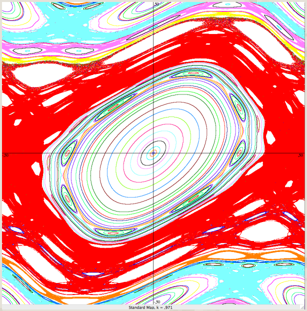
An example: perturbation of a pendulum



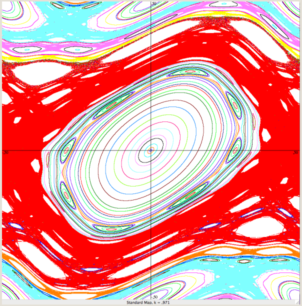


An example: perturbation of a pendulum



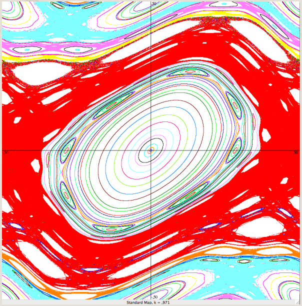


Questions and answers



***Q.:** How many times an orbit visits a subset A in the first N steps?*

Questions and answers



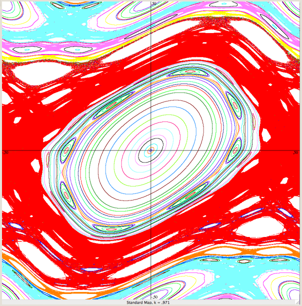
Q.: *How many times an orbit visits a subset A in the first N steps?*

A.: Birkhoff Ergodic Theorem (Law of Large Numbers)

$$\lim_{N\to\infty} \frac{\#\{n = 1, \dots, N \ : \ \text{the orbit of } x \text{ is in } A \text{ at time } n\}}{N} = m(A)$$

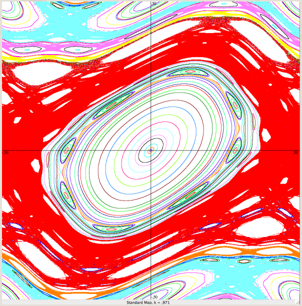
for m-almost every x.

Questions and answers



Q.: *If an orbit has never visited a subset* A *for* N *steps, what is the probability that it visits* A *at time* $N+1$*?*

Questions and answers



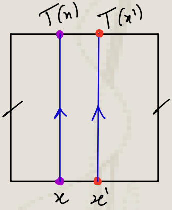
Q.: *If an orbit has never visited a subset A for N steps, what is the probability that it visits A at time $N+1$?*

A.: Mixing or Decay of correlations (Independence of r.v.)

$$\lim_{N\to\infty} m\{x_{N+1} \in A | x_1, \ldots, x_N \notin A\} = m(A)$$

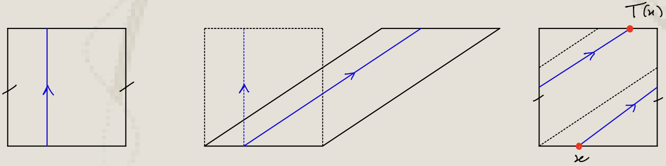
where $x_1, x_2, \ldots$ is the orbit of x.

An example: suspension flows



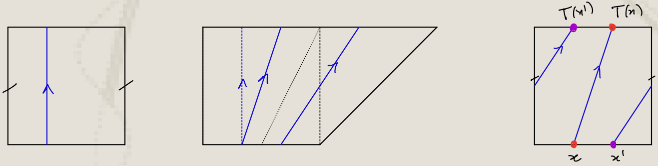
$T : S^1 \longrightarrow S^1 \quad , \quad T(x) = x$

An example: suspension flows



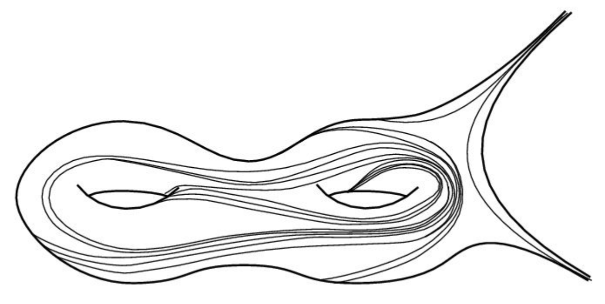
$$T: S^1 \longrightarrow S^1, \quad T(x) = x + \alpha \pmod 1$$

An example: suspension flows



$$T : S^1 \to S^1, \qquad T(x) = 2x \pmod 1$$

Questions



Q.: *How many periodic orbits* γ *exists of length* $\ell(\gamma) \leq T$*?*

Q.: *Speed of decay of correlations?*