



Mathematics Education

Anna Baccaglini-Frank
Dipartimento di Matematica
Università di Pisa



Dipartimento di Matematica
Università di Pisa

‘Research’ or ‘inquiry’?

Some researchers in this field, especially ethnographers, prefer to use the word ‘inquiry’ instead of ‘research’ because the latter is, for some, indissolubly tied to European imperialism and colonialism.

“The word [‘research’] itself is probably one of the dirtiest words in the indigenous world’s vocabulary ...It is implicated in the worst excesses of colonialism [...]

knowledge about indigenous people was collected, classified, and then represented back to the West.[...]

It is so powerful that indigenous people even write poetry about research.”

(Smith, 1999, p.1)

'Paradigm wars'

This term refers to conflicts that occurred around the 1980s, in the context of research and politics, between acceptable types of research (quantitative vs. qualitative).

These conflicts nearly led to the annihilation of qualitative research in education.

At a political and institutional level, postpositivism was favored, which involved the use of experimental methods and government intrusion in the selection of research methodologies.

Keywords in this period:

- Audits
- Efficiency
- High-stakes assessment,
- Test-based accountability
- Science-based research (SBR)



'Paradigm wars'

This term refers to conflicts that occurred around the 1980s, in the context of research and politics, between two conceptual approaches of research (quantitative vs. qualitative). These conflicts were particularly prominent in research in education.

some basic assumptions:

- tests are valid assessments of learning
- quantitative changes in test scores are the direct reflection of teaching quality
- public comparisons of test scores in schools lead to higher quality teaching

At one time

involved
selecting

Keywords in

- Audits

- Efficiency

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- Test-based accountability

- Science-based research (SBR)

12 characteristics of qualitative research

Patton (2002) suggests these 12 key characteristics:

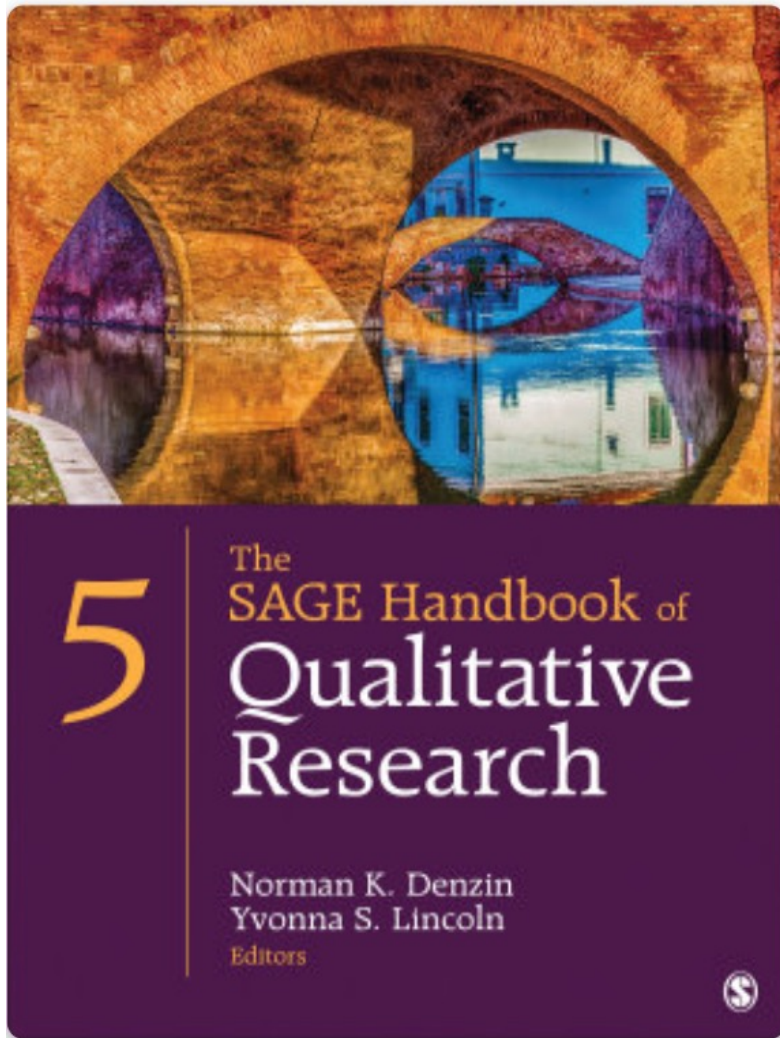
Design strategies

- 1) *Naturalistic inquiry* – The researcher studies real-life situations as they develop, without manipulating or controlling them, being open to whatever emerges (there are no a priori constraints on the results).
- 2) *Emergent design flexibility* – The researcher is open to the possibility of adapting the study as their understanding of the situation evolves and does not constrain themselves with rigid designs, pursuing new exploratory paths as they emerge.
- 3) *Purposeful sampling* – Cases are selected for study (people, organizations, communities, cultures, events, critical incidents...) because they are ‘information-rich’ and illuminating; the sampling is then aimed at studying a phenomenon, not at empirical generalizations to move from the sample to the population.

12 characteristics of qualitative research

Data collection and fieldwork strategies

- 4) *Qualitative data* – Observations that provide dense and detailed descriptions; in-depth studies; interviews that capture the exact words of people, conveying their personal viewpoints and experiences; case studies; detailed document analysis.



2017

Chapter 20 «D...a...t...a..., Data++, Data, and Some Problematics»

Mirka Koro-Ljungberg,
Maggie MacLure, and
Jasmine Ulmer

interesting quotes – «traditional» views

- «this tendency to treat data as “brute” or “given” is traceable to a lingering positivism that lurks in many qualitative studies, even when these studies are committed in principle to interpretive or poststructuralist theoretical frameworks that would fundamentally challenge the notion of a “bedrock” of brute data beneath the “layers” of interpretation or the social constructions of discourse.»
- «data tend to have their moment early in the research process. Indeed, the vocation of data often is to disappear, or at least to recede from view, once it has been marshaled as “evidence,” disciplined into categories, or incorporated into higher-order concepts.»

Koro-Ljungberg, M., MacLure, M., & Ulmer, J. (2017), D...a...t...a..., Data++, Data, and Some Problematics. In N.K. Denzin & Y.S. Lincoln (Eds.), *The Sage Handbook of qualitative research*. SAGE

(Qualitative) Data Dictionary

data “move in and out of aporias, become something different, and resist capture” (Bridges-Rhoads, & Van Cleave, 2013, p. 272).

data “are necessary in empirical research to give evidence or justification for everything you present later on as your findings, such as descriptions, new ideas, relationships between subjects, interpretations and explanations” (Boeijs, 2010, p. 58).

data “generate us as we generate them and produce shards of knowledge that elude categorization” (Nordstrom, 2013, p. 327).

data “are infused with specific circumstances and interests” (Flick, 2014, p. 475).

data “are a source of well-grounded, rich descriptions and explanations of human processes” (Miles, Huberman, & Saldaña, 2014, p. 4).

data “are a joint construction of researchers and participants” (Charmaz & McMullen, 2011, p. 355).

data “are lived and sensed, not merely analyzed.... Data are here and there, and in this space they catch fire, they light up, they become inflamed with desire” (Benozzo, Bell, & Koro-Ljungberg, 2013, p. 311).

data “are the archaeological record of experience” (Bernard & Ryan, 2010, p. 6) or reductions of our experience (p. 5).

data Latin *datum*. Meaning is not ascribed to the object under study but discovered (or given) to the researcher who is able to empirically verify knowledge through the use of the scientific method (Crotty, 1998).

data are a matter of seeing (Schostak & Schostak, 2008).

data are fluid, a chameleon, able to take different “shades” of meaning based on the perspective of the researcher (see, e.g., Hammersley & Atkinson, 1995; Jackson & Mazzei, 2012).

data are “accounts gathered by qualitative researchers” (Polkinghorne, 2005, p. 138).

data and how “data are interpreted and reported will vary significantly depending on the specific epistemological stance undergirding the research process” (Naples, 2003, p. 3).

data typically involve “multiple sources of information” (Creswell, 2013, p. 52).

data “must replicate within the data set” (Lewis-Beck, Bryman, & Liao, 2004, p. 995).

data “can include virtually anything that you see, hear, or that is otherwise communicated to you as while conducting the study” (Maxwell, 2013, p. 97).

data “doesn’t come out however we want” (Barad, 2007, p. 264).

data begin as raw material without inherent meaning until “the interpretive act brings meaning to those data and displays that meaning to the reader through the written report” (Marshall & Rossman, 2011, p. 210).

data from the field “are constructed from talk and action. They are then interpretations of other interpretations are mediated many times over” (Van Maanen, 2011, p. 95).

data guide researchers (Corbin & Strauss, 2008).

data “is tied directly to the trustworthiness of the person who collects and analyzes the data” (Patton, 2002, p. 570).

data, when in the form of human behavior, necessitate “a tolerance for ambiguity, multiplicity, contradiction, and instability” (Wolf, 1995, p. 129).

data are a “vague concept” within the human sciences (Lather, 2012, p. 69).

«We hope to leave readers with the open prospect of unsettlement, discomfort, and uncertainty. Maybe data's different extensions function as discursive apparatuses that regulate diverse effects of power (Foucault, 1980).

Data may not be separated from truth(s), but maybe it is possible to detach the power of truth from its oppressive, controlling, and hegemonic structures and forms.

Can data be deliberated and released from their scientific expectations? Alternatively, if data are seen as concepts or enactments of diverse epistemological connections, then some forms of data are necessarily ontologically fictional and vitally illusive.

Furthermore, data can be seen as a productive illusion or practice that can create movement in researchers, participants, and data's surroundings and political contexts.”

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12 characteristics of qualitative research

Data collection and fieldwork strategies

- 4) *Qualitative data* – Observations that provide dense and detailed descriptions; in-depth studies; interviews that capture the exact words of people, conveying their personal viewpoints and experiences; case studies; detailed document analysis.
- 5) *Personal experience and engagement* – The researcher has direct contact with people and gets close to them, the situation, and the phenomenon being studied. The researcher's personal experiences and understanding are important parts of the study and necessary for understanding the phenomenon.
- 6) *Empathic neutrality and mindfulness* – The researcher adopts a position of empathy during interviews, seeks to understand without judging (neutrality), and demonstrates openness, sensitivity, respect, and awareness. In doing so, they remain fully present and attentive (mindful).

12 characteristics of qualitative research

- 7) *Dynamic systems* – Attention is paid to the process. The researcher expects change, whatever the focus of the research; therefore, the researcher is attentive to the system and the dynamics of the system.

Analysis strategies

- 8) *Unique case orientation* – The researcher assumes that each case is special and unique. The first level of analysis involves respecting and capturing the details of the individual cases being studied; an analysis of multiple cases may follow: this stems from and depends on the quality of the analysis of the individual cases.
- 9) *Inductive analysis and creative synthesis* – The researcher seeks to immerse themselves in the specific details of their data to discover important patterns, themes, and interdependencies. They start by exploring and then confirming, guided by analytical principles rather than rules. The study concludes with a creative synthesis.

12 characteristics of qualitative research

- 10) *Holistic perspective* – The entire phenomenon being studied is viewed as a complex system that is more than the sum of its parts. The focus is on complex relationships and system dynamics, which cannot be reduced to a few discrete linear variables and cause-effect relationships.
- 11) *Context sensitivity* – The researcher places the results in their social, historical, and temporal context and is careful, sometimes doubtful, about the meaning and the possibility of generalizing them to different times and situations. They work with careful comparisons between cases, extrapolating patterns for the potential transferability and adaptability of the results to different contexts.
- 12) *Voice, perspective, and reflexivity* – The qualitative analyst is aware and reflective of their own perspective; a credible voice provides authenticity and reliability. While objectivity is impossible, and total subjectivity undermines credibility, the researcher's focus is on balance, authentically portraying the world in all its complexity while being consciously analytical, politically attentive, and reflective.

Approaches we (at UNIPI) use the most

- case studies
- design-based research
- narrative inquiry & grounded theory

Case Studies

Fundamental underlying question:

“What are the fundamental characteristics of this single case or of these cases in comparison?”

Case study research outlines in fine-grained details one or more cases.

A case is a bounded system (in the sense that it must be clear what belongs to it and what does not), that is, a set of interconnected elements that form an organized ‘whole.’

For example: a child with a learning disability, a student with special educational needs, a specific school, a national program, or even, for some, an event like a protest on a university campus.

Design-based research as a form of use-inspired basic research

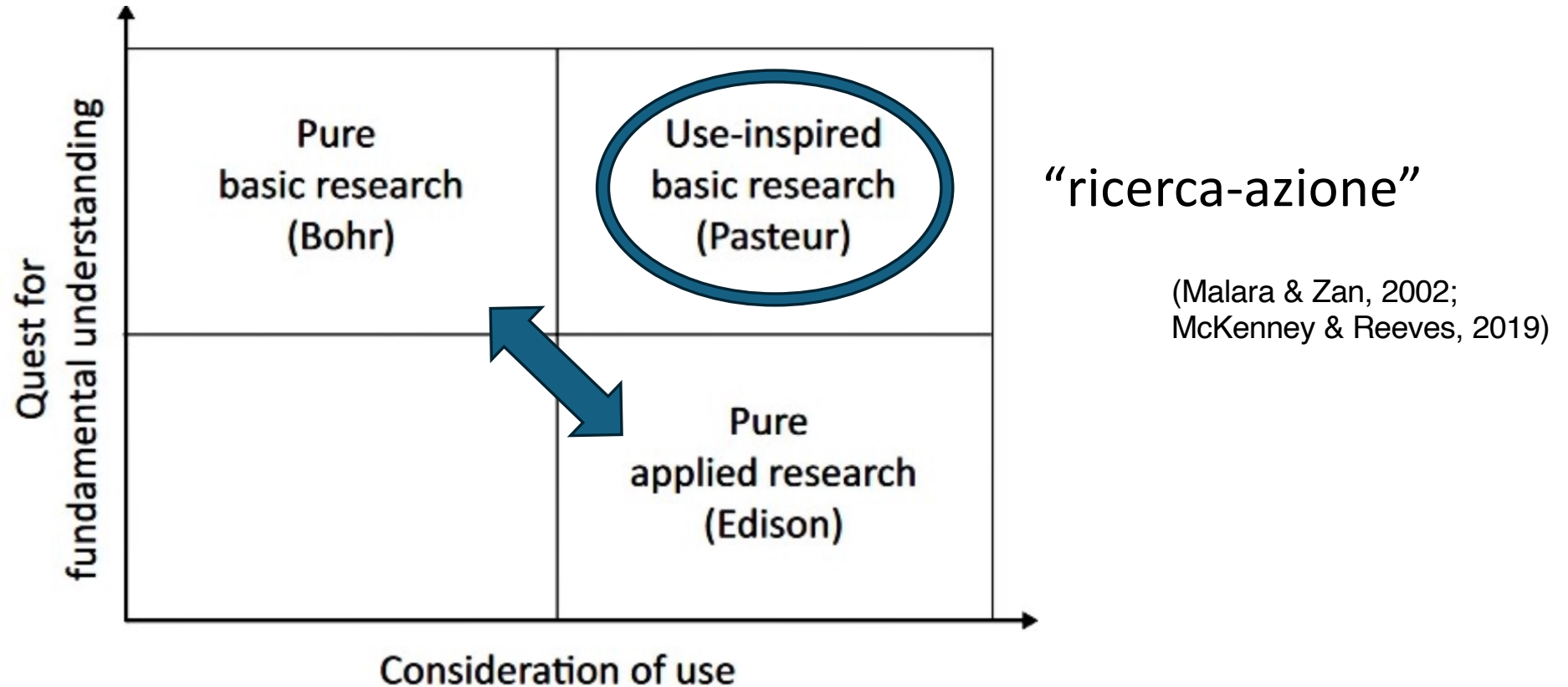


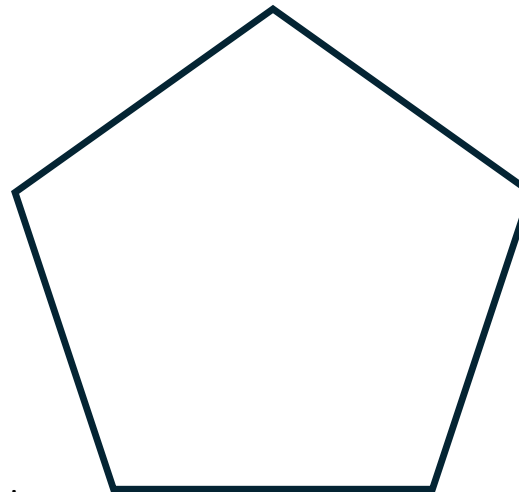
Figure 1.1 Pasteur's quadrant

Source: Stokes, 1997

The “research pentagon”

Research aim:
General direction, problem to which
the research wants to contribute

Research object:
What is to be investigated
and how can it be
conceptualized?

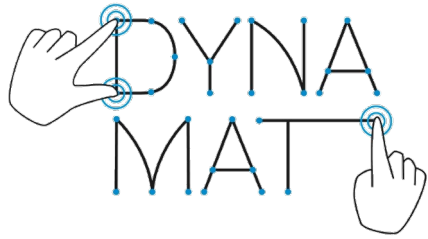


(specific) research questions:
which questions are to
be answered?

Research situations:
external-internal, (external) the discourse in
the field that frames the object and the RQ –
(internal) such knowledge and the methods
must be applied to form the internal situation
for the investigation

Research methods:
which methods (data collection, handling,
analysis) , data analysis, are suitable to lead
to the answers

Bikner-Ahsbabs & Prediger (2006, S. 53), acc. Mason & Waywood (1996). Cf.: Bikner-Ahsbabs (2019)



ongoing
research...

Research aim:

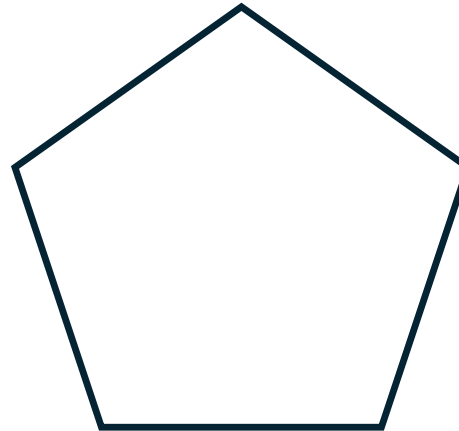
design “effective” digital-
technology-enhanced activities
that support struggling students
(grade 10) in algebra

Interesting phenomenon

Students seemed to be engaging in forms of
discourse with the digital artifacts, and shifting
their discourse in relation to the artifacts’
“responses” (feedback)

Research object:

mathematics technology-enhanced-learning
in the case of students with a history of failure
or low achievement



Research situation:

- struggling students/mathematics learning
difficulties can benefit from appropriately designed
digital means
- remedial learning of algebra
- literature on cognitive and affective/social
dimensions is disconnected
- “deficit discourse” is pervasive and can be harmful

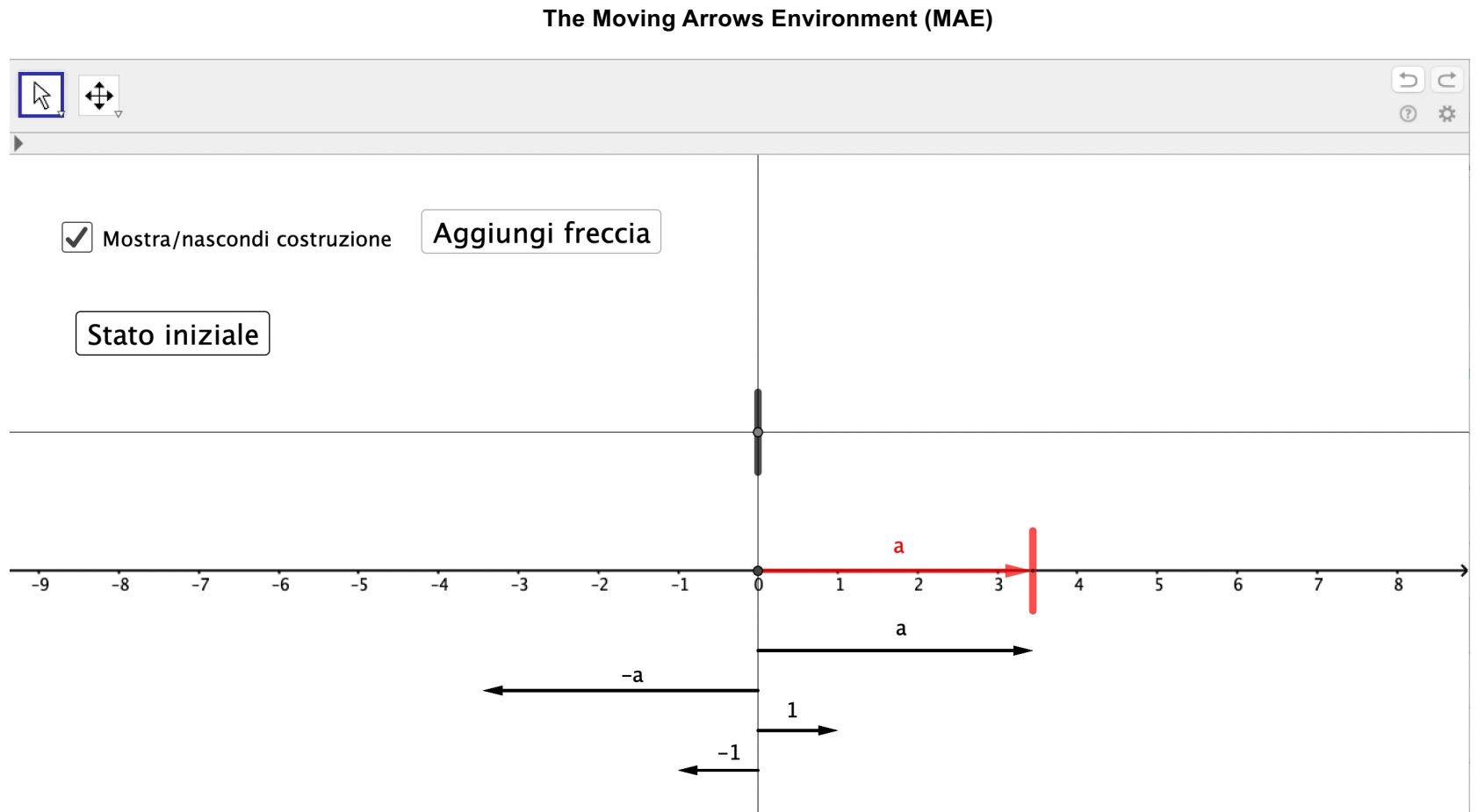
Research method:

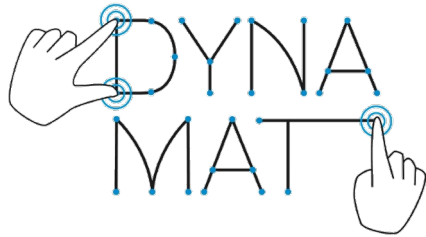
discourse analysis
through the lens of Commognition

The MAE presents a red arrow on a horizontal oriented number line; the endpoint of the arrow is marked with a red tick mark and is directly draggable and bound to the oriented number line. Parallel to the red arrow, there are two black arrows.

The endpoints of the black arrows a and $-a$ move indirectly, one opposite to the other, as the endpoint of the red arrow is dragged, maintaining the same length of the red arrow.

The “Add arrow” button allows the construction of new arrows by creating different consecutive copies (that is arrows with the same length and same orientation) of the black arrows a , $-a$, 1 and -1 on another horizontal line. The endpoint of the last copied arrow is marked with a black tick mark that cannot be directly dragged, but it moves in correspondence with the dragging of the red arrow.





ongoing
research...

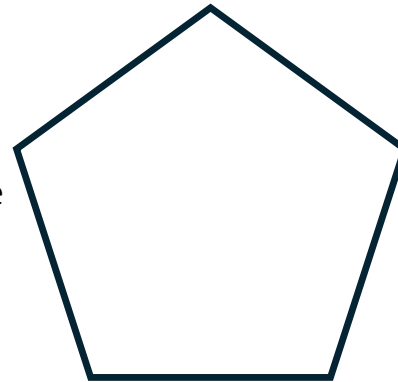
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Research question:

What is the role of the Moving Arrows
Environment in the shift of Cora and Letizia’s
discourses about the expression $-a$?

Research situation:

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difficulties can benefit from appropriately designed
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Research method:

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Discourse alignment

We are interested in capturing possible instances of
a process of *discourse alignment*, that is, a process
through which the discourse produced by A (e.g., a
student) shifts in response to the interactions with B
(e.g., another student or the digital environment), into
a new discourse (produced by A) in which the
narratives produced by B can be endorsed.

Analytic tool

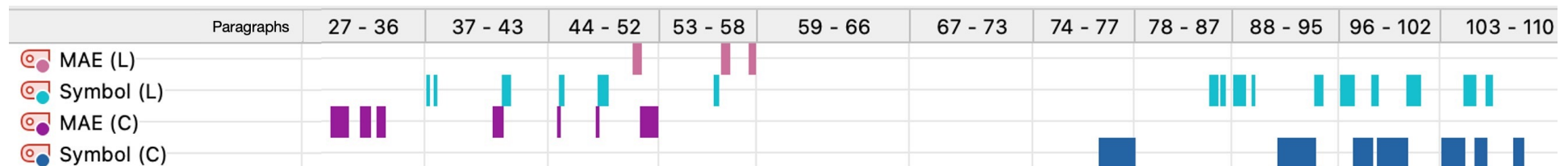
Within MAXQDA, we developed a system to code the speakers' turns in the transcript labelling

“Symbol” → use of algebraic symbols

“MAE” → reference to the digital environment.

This system of labels enables us to highlight which visual mediators are involved in the students' discourse and when they are used. Moreover, these labels are speaker-sensitive (L for Lucrezia, C for Cora).

Therefore, an analysis of their occurrences allows us to notice the differences in the two students' discourse with respect to the visual mediators involved. The visual tool *Codeline* in MAXQDA makes it possible to simultaneously observe all the labels used, where the columns represent ranges of paragraphs of the transcript, while the rows display labels with the mediator used and the speaker.



Coherence and consistency (in research)

- Coherence: compatibility of the various parts in a same whole
 - The theories are important coherent entities in the research
- Consistency: congruity with what has been previously or elsewhere shown or stated
 - consistency is a regularity principle for scientific argumentation

(Lexico, Oxford Dictionary, Marriam-Webster, Mosterin, J. (2011))

Theoretical or conceptual framework & Conceptualizing the research object

Consistency of the argumentation

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(specific) research questions
which questions are to be answered?

Research methods:
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Bikner-Ahsbahr & Prediger (2006, S. 53), acc. Mason & Waywood (1996). Cf.: Bikner-Ahsbahr (2019)

Three types of theories in Math Education

- “A **background theory** is a (mostly) consistent philosophical stance of or about mathematics education ... [it plays] an important role in discerning and defining what kind of objects are to be studied, what constitutes a researchable question, methods, and situation as perceived by the researcher“ (Mason & Waywood, 1996, S. 1058).
- **Framing theory** is used to provide a theoretical frame within mathematics education building on a background theory and knowledge in the field. (Bach et. al. 2020)
- **Foreground theories** are mostly local theories in mathematics education: “....because [of] the foreground aim .. .” , e.g. mistakes in using decimal numbers (Mason & Waywood, 1996, p. 1056).
 - Function of foreground theories may be to **describe** with theoretical terms, allow to **understand** or **explain**, **predict** (if-then-structure, the-the-structure) or **prescribe** (e.g. the value of means for learning a specific topic) (Prediger, 2019)



What is a “theory”?

The field has not reached a complete agreement...

(Assude, Boero, Herbst, Lerman, and Radford, 2008)

A theory



- offers a lens and a language
(Bishop 1992, p. 711, Maier & Beck, 2001, p. 45)
- consists of concepts and claims
(Niss, 2018, p.42-43)
- addresses phenomena (allowing us to see and understand, or explain...)

(Maier and Beck, 2001; Mason and Waywood, 1996)

What do we mean by «theory»?



A theory “consists of

- an *organised network of concepts* (including ideas, notions, distinctions, terms, etc.) and *claims* about some extensive domain, or a class of domains, of objects, situations and phenomena.
- In the theory, the *concepts are linked in a connected hierarchy* (oftentimes of a logical or proto-logical nature), in which a certain set of concepts, taken to be basic, are used as building blocks in the formation of the other concepts in the hierarchy.
- In the theory, the *claims are either basic* hypotheses, assumptions, or axioms, taken as *fundamental* (i.e. not subject to discussion within the boundaries of the theory itself), *or statements obtained from the fundamental claims* by means of *formal or material* (by ‘material’ we mean experiential or experimental) *derivation* (including reasoning)". (Niss 2018, p.42-43)

What do we mean by «theory»?



Radford (2008): “a theory can be seen as a way of producing understandings and ways of action based on:

- A system, **P, of basic principles**, which includes implicit views and explicit statements that delineate the frontier of what will be the universe of discourse and the adopted research perspective.
- A **methodology [methodologies], M**, which includes techniques of data collection and data-interpretation as supported by P
- A set, **Q, of paradigmatic research** questions (templates or schemas that generate specific questions as new interpretations arise or as the principles are deepened, expanded or modified)“ (Radford 2008, p. 320).

Theory= (P, M, Q)

What do we mean by «theory»?



“Results” (R) coming from research may rebound on the theory development:

(Radford, 2012)

$[(P, M, Q), R]$

Ricerca (consistente) consente a una teoria (coerente) di espandersi.

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