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Randomized Two-Sided Gram-Schmidt Process with Applications

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Given two matrices $X, Y \in \mathbb{R}^{n \times m}$ with $m < n$ and full rank, the Two-Sided Gram-Schmidt process aims to find two bases $Q, P \in \mathbb{R}^{n \times m}$ such that $\mathrm{range}(X) = \mathrm{range}(Q)$, $\mathrm{range}(Y) = \mathrm{range}(P)$ and $Q^T P = D$ with D diagonal, i.e. Q and P are biorthogonal. It is widely known that this algorithm frequently suffers from numerical instability, and the bases Q and P are often ill-conditioned.

In this talk, we present a randomized version of the algorithm, which computes two matrices Q and P that satisfy the sketched biorthognality condition $(\Omega Q)^T \Omega P = D$, where $\Omega \in \mathbb{R}^{s \times n}$ is a sketching matrix satisfying an oblivious ε-embedding property, such as a subsampled randomized Hadamard transform or a sparse sign matrix. We show how this approach can improve the stability of the algorithm and the condition number of the computed bases Q and P.

As an application, we consider the computation of approximate eigenvalues and both right and left eigenvectors, where our randomized two-sided Gram-Schmidt orthogonalization process can be implemented within the non-symmetric Lanczos algorithm.

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