

Numerical solution of differential problems with nonlocal boundary conditions

Monday, September 1, 2025 4:10 PM (1h 10m)

We consider a class of differential problems set in a Banach space, with integral boundary conditions:

$$\begin{equation} \frac{dv}{dt} = Av, \quad 0 < t < T, \quad \frac{1}{T} \int_0^T v(t) dt = f, \end{equation}$$

where A is a linear, closed, possibly unbounded operator (e.g., second derivative in space). Note that the finite-dimensional version of this problem, where A is a matrix, is closely related to the task of computing matrix functions $\psi_\ell(A)$, where ψ_ℓ denotes reciprocals of the φ_ℓ -functions used in exponential integrators.

We prove the existence and uniqueness of the solution $v(t)$ and characterize it via a family of mixed polynomial-rational expansions w.r.t. the operator A . From this result we design a general numerical procedure for computing an approximation of $v(t)$ up to a given tolerance. An interesting feature of this approach is the fact that successive rational terms can be computed independently: this allows us to fine-tune the accuracy of the approximation by adding further terms as needed, without having to recompute the whole approximation. Numerical tests focus on a model problem involving a parabolic equation and highlight the effectiveness of this approach.

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Session Classification: Poster Session