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On a class of Petrov-Galerkin Krylov methods for algebraic Riccati equations

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Finding the unique stabilizing solution $X = X^H$ of a large-scale continuous-time algebraic Riccati equation (CARE) $A^H X + XA + CHC - XBB^H X$ with a large, sparse $n \times n$ matrix A, an $n \times m$ matrix B and an $p \times n$ matrix C is of interest in a number of applications. Here, B and C^H are assumed to have full column and row rank, resp., with $m, p \ll n$. The unique stabilizing solution $X = X^H$ is positive semidefinite and makes the closed-loop matrix $A - BB^H X$ stable. Even so A is large and sparse, the solution X will still be a dense matrix in general. But our assumptions on B and C often imply that the sought-after solution X will have a low numerical rank (that is, its rank is $\ll n$). This allows for the construction of iterative methods that approximate X with a series of low rank matrices X_i stored in low-rank factored form. That is, the Hermitian low-rank approximations X_j to X are of the form $X_j = Z_j Y_j Z_j^H$, where Z_j is an $n \times kj$ matrix with only few columns and Y_j is a small square $kj \times kj$ Hermitian matrix. There are several methods which produce such a low-rank approximation.

In this talk, we consider a class of (block) rational Krylov-subspace-based projection methods for solving CAREs. The CARE is projected onto a block rational Krylov subspace $\mathcal{K}_j$ spanned by blocks of the form $(A^H - s_k I)^{-1} C^H$ for some shifts $s_k,\ k = 1, \ldots, j$. The considered projections do not need to be orthogonal and are built from the matrices appearing in the block rational Arnoldi decomposition associated to $\mathcal{K}_j$. The resulting projected Riccati equation is solved for the small square Hermitian Y_j. Then the Hermitian low-rank approximation $X_j = Z_j Y_j Z_j^H$ to X is set up where the columns of Z_j span $\mathcal{K}_j$. The residual norm $\|R(X_j)\|_F$ can be computed efficiently via the norm of a readily available $2p \times 2p$ matrix. We suggest reducing the rank of the approximate solution X_j even further by truncating small eigenvalues from X_j. This truncated approximate solution can be interpreted as the solution of the Riccati residual projected to a subspace of $\mathcal{K}_j$. This gives us a way to efficiently evaluate the norm of the resulting residual. Numerical examples are presented.

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