

A subspace-conjugate gradient method for linear matrix equations

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Large-scale multiterm linear matrix equations of the form

$A_1XB_1 + \dots + A_\ell XB_\ell = C$, arise as the algebraic formulation in various application problems, such as discretized multivariable

time PDEs and inverse problems. While effective methods exist for two-term equations ($\ell = 2$), limited options are available for $\ell > 2$. Thus, efficiently solving multiterm matrix equations remains an open problem in numerical linear algebra. In this talk, we present a new iterative scheme called the Subspace-Conjugate Gradient (Ss-cg) method for the efficient solution of large-scale multiterm linear matrix equations. This method relies on the matrix-oriented CG scheme but better uses the underlying (low-rank) matrix structure. By imposing a peculiar orthogonality condition, the CG scalar coefficients for the iterative solution and the descent direction are replaced by low-dimensional matrices in Ss-CG. We employ truncation strategies to maintain the computed matrix iterates of low rank since limiting memory consumption becomes essential, especially when the number of terms ℓ is large. To this end, an additional ad-hoc randomized range-finding strategy is developed to further speed up computations.

The features of the Ss-CG method lead to remarkable computational gains, as demonstrated by several computational experiments. In particular, we compare Ss-CG with existing algorithms for solving Sylvester and Lyapunov multiterm equations to highlight its potential.

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