Contribution ID: **10** Type: **Talk**

Preconditioning Strategies for the Riesz Operator by Rational Approximations

Wednesday, September 3, 2025 11:30 AM (30 minutes)

In this talk, we investigate preconditioning strategies for the Riesz operator $-(-\Delta)^{\frac{\alpha}{2}}$, $\alpha \in (1, 2]$, commonly used in fractional models such as anomalous diffusion. For α close to 2, it is well known that the Laplacian itself serves as an effective preconditioner with linear computational cost. However, as α decreases toward 1, its performance deteriorates, requiring more specialized approaches.

Our approach consists of the following steps:

- approximate $-(-\Delta)^{\frac{\alpha}{2}}$ as the fractional power of a given discretization of the Laplacian, using the Matrix Transfer Technique;
- represent the resulting fractional power operator in integral form, adopting the Dunford-Taylor integral representation;
- explore various quadrature rules to approximate the integral, leading to rational approximations of the fractional power operator.

As a result, we define and analyze preconditioners for the Riesz operator, expressed as a sum of m inverses of shifted Laplacian matrices, where m corresponds to or is closely related to the number of quadrature points.

Using a Gauss-Jacobi quadrature approach, we show that while the resulting preconditioner performs comparably to the Laplacian for α close to 2, it yields significantly better results for α close to 1, even with a reasonably small m, still retaining the same computational complexity as the Laplacian. To further enhance these results, we investigate the feasibility of using exponentially convergent quadrature rules, like sinc and Gauss-Laguerre, to ensure the number of inverses remains small while achieving numerical optimality.

Primary authors: ACETO, Lidia (University of Eastern Piedmont); MAZZA, Mariarosa (University of Rome Tor Vergata)

Presenter: MAZZA, Mariarosa (University of Rome Tor Vergata)

Session Classification: Morning Session