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On the well-posedness of the best low-rank tensor approximation problem

Thursday, September 4, 2025 9:00 AM (30 minutes)

The canonical polyadic decomposition (CPD) of a higher-order tensor is its basic decomposition in a minimal sum of rank-1 terms. CPD plays a major role in data analysis and signal processing by allowing for unique recovery of underlying factors. However, it is well known that the low-rank approximation problem is ill-posed in general. That is, a tensor may fail to have a best rank-R approximation when $R > 1$.

In this talk, we switch to the opposite point of view, namely that the CP approximation problem is well-posed, unless the tensor is "too far"from the tensors with rank at most R. This is more an engineering perspective: the problem is well-posed unless the data are too noisy for a proper analysis. We give deterministic bounds under which the existence of a best low-rank approximation is guaranteed [1, 4]. In the derivation of the bounds, a tensor is viewed as a linear but multi-matrix pencil. We develop a computational strategy in which from each pencil only numerically reliable information is obtained (i.e., invariant subspaces rather than eigenvectors if the latter cannot be estimated reliably), and the results combined. We propose new CP estimation schemes that rely on such eigenspace computations [2, 3].

References

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2. Eric Evert, Michiel Vandecappelle, Lieven De Lathauwer, A Recursive Eigenspace Computation for the Canonical Polyadic Decomposition, SIAM Journal on Matrix Analysis and Applications, 43 (1), 2022, pp. 274–300, https://doi.org/10.1137/21M1423026.
3. Eric Evert, Michiel Vandecappelle, Lieven De Lathauwer, Canonical Polyadic Decomposition via the Generalized Schur Decomposition, IEEE Signal Processing Letters, 29, 2022, pp. 937–941, https://doi.org/10.1109/LSP.2022.3156870.
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