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Low-Rank Numerical Solution of Ordinary and Fractional Differential Equations via the ⋆-Approach

Thursday, September 4, 2025 3:00 PM (30 minutes)

We study the numerical solution of non-autonomous linear ODEs of the form d$\frac{}{dt\tilde{u}(t)=\tilde{A}(t)\tilde{u}(t), \quad \tilde{u}(a)=v,} where$(t) $\in \mathbb{C}^{N\times N}$ is analytic and often takes the form (t) = $\sum_{j=1}^{k} A_j f_j(t), with large, sparse constant matri$
and scalar analytic functions $f_j(t)$. Such equations commonly arise in quantum chemistry, particularly in spin dynamics. In general, no closed-form solution exists, so we proposed a spectral method—called the **⋆-approach**—which expands the solution in Legendre polynomials and approximates the coefficients by solving a structured matrix equation: X - F$_1 X A_1^T - \cdots - F_k X A_k^T = \phi v^T, where the matrices$ F$_j$ encode the functions $f_j(t)$ in a structured algebra. This equation has favorable properties: (i) banded F_j, (ii) Kronecker-structured A_j, and (iii) a rank-1 right-hand side. These allow efficient iterative and low-rank methods.

We implemented this strategy in the so-called ⋆-method, shown to be highly efficient for the generalized Rosen-Zener model, with linear scaling in system size and strong performance on large time intervals (collaboration with Christian Bonhomme - Sorbonne University, and Niel Van Buggenhout - Universidad Carlos III). Preliminary results also show promise in extending the method to fractional ODEs, where similar matrix equations arise and exhibit low-rank structure (collaboration with Fabio Durastante —University of Pisa, and Pierre-Louis Giscard —ULCO).

Primary authors: BONHOMME, Christian (Sorbonne University); DURASTANTE, Fabio (Università di Pisa); VAN BUGGENHOUT, Niel (Universidad Carlos III); POZZA, Stefano; GISCARD, Pierre-Louis (Université Littoral Côte d'Opale)

Presenter: POZZA, Stefano

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