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On block Krylov and matrix polynomials

Friday, September 5, 2025 10:00 AM (30 minutes)

The deep connection between Krylov methods, scalar orthogonal polynomials, and moment matrices is well established, especially for Hermitian and unitary matrices. In this talk, we consider the extension of this framework to block Krylov methods and orthogonal matrix polynomials.

By representing elements of a block Krylov subspace through matrix polynomials, we consider the matrix-valued inner product introduced in [1] that, under non-degeneracy, defines a linear isometry. This yields a one-to-one correspondence between orthonormal matrix polynomials and orthonormal bases of the block Krylov subspace.

For normal matrices, the block Gauss discretization [1,2] of such an inner product admits an integral representation familiar from the theory of orthogonal matrix polynomials. As an application, we extend a Szegő-type short recurrence [3] to the block Arnoldi algorithm applied to unitary matrices.

Finally, we analyze the structure of the block moment matrix and explore its relation to orthogonal matrix polynomials and recurrence coefficients via a Cholesky-like factorization.

1. Lund, K.: A New Block Krylov Subspace Framework with Applications to Functions of Matrices Acting on Multiple Vectors, https://www.proquest.com/docview/2046209015?sourcetype=Dissertations%20&%20Theses, (2018)
2. Zimmerling, J., Druskin, V., Simoncini, V.: Monotonicity, bounds and acceleration of block Gauss and Gauss-Radau quadrature for computing $B^T\phi(A)B$, https://doi.org/10.1007/s10915-025-02799-z, (2025)
3. Sinap, A., Van Assche, W.: Orthogonal matrix polynomials and applications, https://doi.org/10.1016/0377-0427(95)00193-X, (1996)

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