RETHINKING COGNITIVE UNITY OF THEOREMS IN THE PERSPECTIVE OF HABERMAS' RATIONALITY

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In my presentatation I will deal with cases of failures and success in achieving cognitive unity of theorems, as a paradigmatic situation to show how Habermas' construct of rationality may be used:

- As an analytical tool **to identify the reasons of failure**;
- As a tool to enable students **to be succesfull**

and also

- As a tool to put into evidence similarities between achieving cognitive unity of theorems, and ways of attaining the aims of an investigation in other scientific domains (**an emerging possibility of transdisciplinarity**)

In doing it, reference will be made to the important role played by the **collaboration** with **Maria Alessandra Mariotti** in two important moments of the evolution of my research – and also to **hints** and **encouragements** received by two other protagonists of these days: **Nicolas Balacheff** and **Ferdinando Arzarello**.

THE ROOTS OF THE CONSTRUCT OF COGNITIVE UNITY OF THEOREMS

Spring 1995: **Rossella Garuti** asks her 8-grade students ***if it is possible that two non-parallel sticks produce parallel shadows, and to justify their answers*** (the idea of this task came from a question posed by a student in a previous classroom discussion). Since grade 6, students were familiar with producing hypotheses and trying to justify them (the field of experience of Sun shadows had offered many occasions for it).

Something «strange» happens: students engage in exploration by imagining to look at two parallel shadows on the ground, then they look at two sticks in their hands «from the Sun», or «in front to the Sun», with their eyes that look at the imagined shadows on the ground and then move towards the imagined Sun, then they imagine beams of parallel sunrays "slipping" on parallel planes containing the non-parallel sticks, then they «see» this as the **condition for parallelism of shadows**, then they arrange a **justification** for the conjecture by **coming back** to the idea of parallel sunrays that draw parallel shadows by «slipping» on the parallel planes that contain the sticks, etc.

The whole activity results in individual texts that reflect personal ways of moving back and forth between shadows and the sun through the sticks, up to a **rather well organized (in several cases) validation of the conjecture.**

Rossella **senses** that this «strange» thing may be a case of a general phenomenon of interest for mathematics (and for mathematics education). Through discussions we arrive to express such phenomenon in terms of the **possibility that, for some conjectures (**or already known statements)**, proof may be constructed by selecting and arranging in a standard proof: arguments, facts, inferences emerging from conjecturing** (or from the exploration of the content of the given statement). A possible, accessible way for actively approaching proving by students! But… how to develop the emerging idea of «cognitive unity of theorems» in a **theoretical perspective** concerning theorems and proofs, with related cognitive aspects?

Maria Alessandra was working on this perspective… Enrica Lemut was working on strategic thinking…Already in the past I had occasions to collaborate with Maria Alessandra… Brief: in the 1995 summer we produced **two PME-XX research reports** (signed by Boero, Garuti, Lemut & **Mariotti**, and by Boero, Garuti & **Mariotti**) on the cognitive (strategic) and on the mathematical and educational sides of «cognitive unity of theorems» (more than 450 citations up to this moment). And this construct entered the **PME-XXI Research Forum** (Bartolini, Boero, Ferri, Garuti & **Mariotti**), which was also the first occasion for Maria Alessandra to present the **construct of Theorem as a Statement validated by a Proof in the context of a Theory.**

This was the origin of the construct of cognitive unity of theorems**. It was not born in mathematics, it was born in the field of experience of Sun shadows:** we will come back to this point later!

THE ROOTS OF THE ADAPTATION IN THE FIELD OF MATHEMATICS EDUCATION OF HABERMAS' ELABORATION ON RATIONALITY

My visit in Eritrea in the year 2000 was a tourning point for my research interests. In particular (but not only!) the discovery of a way of interpreting Sun shadows in terms of a dynamic equilibrium between light and darkness, with the possibility by seventh graders to solve in a non-geometrical way most of the problems that were not easy to solve for our students by using the geometrical model, was the crucial occasion for me to try to answer a complex question that sometimes I had already encountered and then abandoned (under the pressure of developing a standard kind of research in mathematic education): **How to identify and compare different ways of thinking and acting in the world, which are rooted in different cultures?**

In those years I had several occasions to meet **Nicolas Balacheff**, due to the Ph. D. theses of Bettina Pedemonte and Elisabetta Robotti in Grenoble. I asked him if his cKc construct could have been a suitable tool to cope with my new research interests. After two discussions at some distance of time, Nicolas told me: **NOT, it is better that you read Habermas' text on rationality!**

My first encounter with Habermas' text was not very productive; the complexity of the discourse and the condensed style of presentation were hard to access for me. The only thing that I was able to derive from that reading was the idea of a «dynamic» conception of rationality, embedded (on the epistemic and on the communicative side) within a culture.

In the same months I read a paper on rationality written by Alain Lerouge and published on RDM. My reaction to Lerouge's narrow and static perspective brought me to re-read Habermas (not only the text suggested by Nicolas, but also some previous texts, on communicative rationality, in order to understand the former one). By this way I started to understand Habermas!

In 2003, **Ferdinando Arzarello** invited me to present my ideas on Habermas' rationality and its possible implications in mathematics education in a seminar in Turin. It was **an important occasion for me** to connect rationality with proving and to elaborate some **«germs» of three working hypotheses** concerning rationality, which **now** I can formulate this way: **The rationality construct can be used:**

- **In mathematics**, as an effective analytical tool to identify several reasons of failure in proving and in math. modelling (in terms of lack of rationality on its epistemic or teleological or communicative sides)
- **In mathematics**, as an effective tool to plan teaching in order to develop students' rational behavior in proving and in mathematical modeling.
- As a tool to **compare the rationalities of different disciplines, and of different domains within a discipline**.

My first paper on Habermas' rationality was a PME research report in 2006 and had a very ambitious title:**Habermas' theory of rationality as a comprehensive frame for conjecturing and proving in school**. Other papers followed, most of them in collaboration with Francesca Morselli or Nadia Douek or Elda Guala, and more recently with Fiorenza Turiano.

For about 10 years, the work on Habermas' rationality in math. education concerned only the people that engaged with me in the joint use and development of the construct, mainly in the case of proving. Then the situation changed, and in the last three years several papers were published on the use (and further elaboration) of the Habermas' construct in different directions: as a tool to inform teachers' actions to develop students' argumentative skills ("**rational questioning"**); as a tool to develop critical thinking in basic statistical education; as a tool to deal with the use of CHATGPT in the classroom; etc.(among them, 6 papers published on outstanding journals in mathematics education and in science education).

Concerning **Maria Alessandra**, for at least fifteen years I was not able to involve her in a collaborative relationship on the adoption of the **rationality construct in mathematics education**. The difficulty to move from Habermas' philosophical and sociological perspective (and terminology) to an autonomous, consistent and coherent transfer of his elaboration in the field of mathematics education and the difficulty to provide evidence for its utility in the case of proving were sufficient reasons for my difficulty of communicating with her. But… **after CERME-13 in 2023**, a new, important occasion of collaboration was her careful, critical-constructive reading of the paper by Nadia Azrou and me, with a lot of precise comments. Here are two examples of her comments:

- <…problem solving situations concerning proof>*Quali? Non è chiaro. problemi aperti di congettura? Problemi chiusi del tipo prove that ...A seconda della tipologia di compito / problema cambiano i processi e di conseguenza anche la consapevolezza*
- <and the level (of knowledge and of meta-knowledge) on which rationality was exercised>. *Questa espressione (esercitare la razionalità) non mi è chiara . Se è possibile osservare un comportamento (cosa fa o scrive un soggetto) e analizzarlo secondo criteri dati a priori, non so come osservare l'esercizio della razionalità perché in questo modo si prende la prospettiva del soggetto che agisce e si assume una intenzionalità che possiamo classificare come razionale nell'azione svolta.*

She also sent to me a partly alternative, detailed interpretation (in terms of the Habermas' construct) of one of the students' productions! **Some of her comments (see above) opened for me a more general perspective,** particularly as concern **method issues**, and the possibility of **broadening the scope of the use of the Habermas' construct** (thus coming back to my original motivation for Habermas' rationality). This is one of the reasons why I will spend the second part of my intervention to illustrate some new ideas that connect the use of H. rationality for proving in the case of **cognitive unity of theorems** with a **more general perspective of «cognitive unity»** that concerns key activities in very different cultural domains – a possible perspective, also, to prevent the cultural specificity and strength of the disciplines to be abandoned, as it happens in the current design of many STE(A)M interdisciplinary activities

LET US CONSIDER NOW HABERMAS' RATIONALITY IN THE CASE OF PROVING

A SELECTION OF ESSENTIAL FEATURES OF THE VERY DENSE HABERMAS' ELABORATION:

a discursive practice is considered "rational" when the subject intentionally:

- **ER**: Justifies her claims according to shared principles in a given culture (**not necessarily** succeeding in it)
- **TR**: Adopts strategies to achieve her aims according to past reflected experience (**not necessarily** achieving them), and asks herself why they were effective of not in the new situation
- **CR**: Chooses communication means, in order to reach the interlocutors and enter a shared "space of communication" with them (**not necessarily** achieving such goals) - the interlocutor may be the subject herself in the inner dialogue

NOTE: "**not necessarily**" is an important reason for the adaptation of HR in educational studies, if education is conceived as a dynamic, evolutive process.

ADAPTING IN-DEPTH ASPECTS OF HABERMAS' ELABORATION TO THE CASE OF PROVING (and, in recent literature, to defining, to mathematical modelling, to problem solving in general, to teacher's activities in the classroom, to the relationship with AI, to basic education in statistics) **IS BASED ON A SALIENT ASPECT OF HABERMAS' ELABORATION ON RATIONALITY:**

the evolution of a rational discourse relies upon the intertwining of the three components ER, TR and HR.

- AN EXAMPLE (in the case of an individual proving)*:*

the intention of justifying an intermediate claim ***(ER),*** *when the justification is not immediately available (for instance, by referring to a postulate or an already proven statement), may need the adoption of suitable strategies* ***(TR)*** *to achieve a justification (possibly retrieved from previous reflected experience), and an effective inner communication with herself, supported by suitable semiotic tools* ***(CR),*** *whose inner correctness and correct fitting* ***(ER)*** *with the aim to be achieved must be checked.*

LET US CONSIDER NOW THE CASE OF THEOREMS FOR WHICH THE COGNITIVE UNITY COULD BE ACHIEVED AND FAILS TO BE ACHIEVED BY SEVERAL (OR MOST) STUDENTS

(i.e. the case of proofs that could be constructed by exploiting arguments and knowledge about the situation, produced in the conjecturing phase or during the exploration of the statement to be validated: Boero, Garuti & Mariotti, 1996), but students fail to exploit the *information*, derived from exploration, to get a *valid* proof. It happens for different reasons:

- Those of **structural character** investigated by Pedemonte (2007) at the high school level (e.g: blockage in the transition from abductive or inductive arguments, to deductive organization of proof)
- Those inherent in the **lack of mastery of the epistemic requirements of proof**, like in some episodes at the university level in an Italian university, reported in Azrou & Boero (2024)
- Still in the same Italian university, those identified by Boero (2025), depending on the **presumed necessity of producing an algebraic** – **analytic proof** in elementary number theory and on the difficulties and pitfalls when moving from semantic arguments resulting from exploration, to formal-syntactic arguments for proof.
- Those resulting from the adoption of unsuitable strategies of exploration, **without any reflection on the reasons** for their choice and (a posteriori) for why they were ineffective

In order to appreciate the descriptive and interpretative power of HR in the perspective of elaborating educational strategies to develop students' competencies in the case of cognitive unity of theorems, let us consider a task that resulted in a failure of the majority of students of all levels, and a task (**in geometry**) where a situation like those described by **Pedemonte** happened.

IN ELEMENTARY THEORY OF NUMBERS:

In grades VIII, IX, X , etc. till to grade XVII (master degree in mathematics, chosen curricular option: mathematics education, after the previous 3 years in common) this task was proposed:

- **What is the GCD of all the products of three consecutive natural numbers?**

Some examples at the university, **Master Degre level in Mathematics** (See Boero, CERME 14)

A) numerical cases, with the discovery that the GCD might be 6. The exploration of numerical cases is immediately abandoned (without wondering why the emerging GCD is 6: no sign of TR) to move to the algebraic representation of the situation, in order to get an algebraic proof (in the subsequent interview: **"because I must get a rigorous proof",** lack of ER on the goal to achieve)

Two examples of what follows show a lack of mastery of ER, concerning quantifiers:

$n(n+1)(n+2)=\ldots=n^3+3n^2+2n=KD$ OK, the GCD is 6 because if $n=1$ I may choose $K=1$ Another students moves to an analytic representation with a polynomial function and parallel lines: ***$y=x^3+3x^2+2x$; $y=KD$*** and gets the same conclusion (lack of ER check)

B) Like A) as concerns the presumed necessity of an Algebraic strategy , with strange work on algebraic expressions, for instance**:** ***$x(x+1)(x+2)=x^3+3x^2+2$,*** then the division: ***$(x^3+3x^2+2):(x-6)$*** with no ER control and no TR reflection; then chaotic exploration with numerical cases and the use of Euclid's theorem follows, up to the end of the time.

C) Some students (before or after the conjecture GCD=6) behave like this student:

** Number line 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14,….*

X X X X X X X

V V V V

- *"2 comes each two numbers (odd-even), 3 comes each three numbers"*
- *"4x5x6= 120 is divisible by 6 9x10x11=990 is divisible by 2x3 2 and 3 are common factors.* ***I am unable to go on*** *"*

The student is stuck, no reflection follows on what the three ways of representing the situation should, and could, communicate (TR+CR); and no reflective connection is established between them (TR)

In another similar case, after a while the student writes: ***(n-1)n(n+1)*** with subsequent, different algebraic transformations, without any reflection and connection (TR) with his previous exploration of the number line, which might have suggested an effective use of such algebraic representation.

IN GEOMETRY (end of grade X)

Given a triangle, is it always possible to find a crf that is tangent to the three sides of the triangle?

One of the typical reasonings is as follow:

* *Which are the properties of such a crf?* (he draws a sketch, CR). *Yes, I see: it must be tangent to the three sides of the triangle. Yes, we have seen that it happens inside one angle, with the centre on the bisector.* ***To get a triangle****…(TR) I draw two tangent non parallel lines* (drawing), *I get two sides and I add a third tangent non parallel line, it is the triangle. I have proven it , it means that the centre must be in the intersection of the bisectors of the three angles* ***of a triangle.*** *I think that it is true that, given a triangle, (etc*.)

A serious problem emerge with this student about the lack of ER control on the hypothesis-thesis relationships. **The abductive reasoning is not exploited to get a valid proof**.

* A few students, after the exploration, go on like this student: *But the triangle that I have found is the given triangle? It might be another triangle. No, I did not prove what I had to prove. Thus, if I start with the given triangle, …*

Then this student is stuck: the lack of mastery of TR prevents the student from exploiting the information on the situation resulting from the exploration.

By comparing the above situations and many others, the following critical points in students' proving emerge in specific moments/phases:

A) **Lack of ER both at the mathematical level** (mastery of concepts, including definitions and related quantifiers) **and at the meta-mathematical level** (concerning what proof and proving means, and the relativity of proof validity according to the chosen theory – *see Mariotti's definition of "theorem" as a statement, a theory and a proof that must satisfy the epistemic requirements of the theory*). Vergnaud's definition of «concept», <*at present under investigation by Azrou in the perspective of an extension to the «concept of proof»*>, looks as a promising analytical tool to deal with the lack (and the remediation in the future) of ER – see Azrou & Boero, CERME-13

B) **Lack of TR** as concerns reflections on strategies to adopt and on adopted strategies:

C) **Lack of CR**, particularly as concerns the production of algebraic and geometric signs

AN ALTERNATIVE SITUATION: Here I propose an example (not an isolated case, in his classroom – **after one year and one half of systematic teacher's guide and support of students' rational behavior**) of a grade X student who deals with the same problem considered before:***What is the GCD of all the products of three consecutive natural numbers?***

(a strategy of numerical exploration starts - TR)

4x5x6=240 The divisors are 2,2,5,2,3 8x9x10=720 The divisors are 2,2,2,3,3,5,2

16x17x18= 2,2,2,2,17,2,3,3 too many equal divisors, I see nothing ***(TR evaluation →strategy refinement)***

11x12x13= The divisors are 11, 3, 2,2,13. What happens with other consecutive prime numbers? 29x30x31: the divisors are 29,3,5,2,31. Uhm… 3 and 2 are always there. Why? **(a legitimate abductive move, ER+TR)**

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11,12,13, 14, 15, 16, 17, 18

OK, each 2 numbers there is one multiple of 2, and the multiple of 3 follow each one with a jump of 3. Each three numbers, one is divisible by 3 and one is divisible by 2. ***(ER)*** *Only one divisible by 2* ***(ER)****? Not, on the line* ***(TR)*** *I see: one or two are divisible by 2.* ***(ER)*** *Is this a valid proof? We proved the sum of two consecutive odd numbers divisible by 4 with algebra, and also the product of two consecutive even numbers divisible by 8. Let us try* ***(TR):*** *n.(n+1).(n+2)=……… $n^3+3n^2+2n=n(n^2+3n+2)$. I see nothing* ***(TR/ev)***

Another way ***(TR)*** *might be more useful: (n-1)n(n+1)=…….n^3-n=n(n^2-1). I see nothing.****(TR/ev)***

But I have seen that each three numbers. Yes ***(TR):*** *n(n+1)(n+2): n or n+1 are divisible by 2.*

Why? ***(ER)*** *If n is odd, n+1 is even. And the same for 3: n not divisible by 3? n+1 or n+2 are divisible by 3.* ***Why?*** ***(ER)*** *The rest of the division may be only 1 or 2. And why 6 is the GCD, the <u>greatest</u> common divisor? Obvious: 1.2.3 =6*

NOTE: **why** was probably related to previous experiences, like that with **2n(2n+2)=4n(n+1)** to get the divisibility by 8 in a rigorous way

Then the student builds up a standard proof text by using the elements produced during the exploration:

The GCD of all the products of three consecutive natural numbers is 6. The GCD cannot be greater than 6 because 1.2.3=6. I write the general product of three consecutive natural numbers this way: n(n+1)(n+2). Either n or n+1 is divisible by 2 (due to the alternance odd-even natural numbers). If n is divisible by 3, n(n+1)(n+2) is divisible by 3.2=6. If not, the remainder of the division on n by 3 is 2 or 1. In the first case n+1 is divisible by 3. In the second case n+2 is divisible by 3. Again: 3x2=6 is a divisor of n(n+1)(n+2). Thus 6 is the GCD of all the products of three consecutive natural numbers.

Such rational behavior is not a spontaneous behavior of an exceptional student. It results from a theoretically grounded **teaching strategy**: the teacher had decided:

- at which level of rigour to mediate proving in the arithmetic domain

(the level of an algebraic proof relying upon semantic considerations in the perspective of algebraic modelling (see Morselli & Boero, 2012) – thus establishing the **epistemic constraints for proving**, with reference to **Maria Alessandra**'s construct of **Theorem**)

- which role to assume in the classroom activities (the role of mediator of rationality, through systematic rational questioning – see Conner's work, from CERME in 2017 to Zhuang & Conner, ESM 2024), **in the Vygotskian perspective of internalization of rationality** - **but something is still lacking, in this perspective… And here, Maria Alessandra,… help us!**

The research is now developing through two lines of experimental design:

- (still in grade 10) **The students gradually assume the roles of ER, TR and CR mediators** through Salerno' storytelling, with a deep, NECESSARY change of students' assessment

  (the ordinary assessment being incompatible with the development of rational behavior!)

- After have assumed the roles of mediators of rational behavior in previous years (see the book «Oltre noi e il sole»), teachers **mediate the roles of ER, TR and CR mediators** (in two grade V classes) **in a direct way**, starting by posing questions like: «*Lucia, what would I ask in this moment on Mario's claim?*» «*Stefano, what would I ask in this moment on Lisa's strategy?*» etc. in the perspective that students themselves **adopt rational questioning in their peer interactions** (some encouraging examples already emerge from transcripts!)

(like a dream)

A common, core feature (perhaps, an **intervention model in the future**!) of these different long term experiments is the **dialectic relationship** that develops and involves the poles of students' discursive actions, evaluated by the teacher in the HR perspective, and the teacher's rational questioning purposefully focussed on the aspects of students' HR to be promoted.

Such dialectic development demands and at the same time favours:

- the teachers' progressive growing up as **rational agents of change** in the classroom (what in the reality had been **lacking, behind those cases of failure in achieving the cognitive unity of theorems**, even at the university level!)
- the development of the students' reflective competencies on their own activities, in the ideal perspective of becoming **rational interlocutors among them, and with the teacher** (Radford's "joint labour")

AGAIN: the productive functioning of such dialectics is not compatible, on the teacher's and the students' side, with the usual kinds of students' assessment by the teacher, based on "exhibitionistic" in→out alienating performances under the teachers' requests.

OUTSIDE MATHEMATICS…

Now I would like to move in another direction by considering one **personal experience** that I may interpret **IN THIS MOMENT**, thanks to the evolution of the studies on the adaptation of HR in mathematics education, as **a case of cognitive unity in the field of medical sciences**, where the difficulties met and the advancement realized look rather similar (under the lenses of Habermas' Rationality) to what happens (better: **should happen**) in the case of cognitive unity of theorems.

At the end of the seventies I was involved in a collaboration (as a mathematician with some competencies in the field of statistics) within a team of researchers of the Genoa Research Institute on Cancer (IST).

In the last weeks I was able to retrieve, from my collection of documents, drafts, notes, etc., the field notes and some transcripts of what happened during that one-year long collaboration.

The IST researchers were dealing (inside an international collaboration) with the problem of the growing number of chemical products, and the related increasing risk of diffusion of cancerogenic substances. In- vivo tests demanded too much money and time and also involved ethical issues concerning animals (a warm debate in those years). **Thus in- vitro tests had to be designed and compared with in-vivo tests.**

A lot of information had to be collected (**interesting results of previous studies to be identified and selected**, new **exploratory studies** to be planned (**TR**) and analysed (**ER**), etc). Specially at the beginning, I was ignorant about scientific and technical issues, but probably this ignorance allowed me to appreciate what favoured the gradual progress of the research and the overcoming the obstacles met (together with my gradual entering in the problem dealt by them). Previous experimental strategies were analysed and adopted or abandoned according to evidence of effectiveness AND the search for the reasons why they had been effective (or not) (**TR**). Still I remember the head of the team who repeated *«if we do not realize why that low cost experiment resulted in a failure, we risk to pay one hundred more money for a future experiment with the same pitfalls».* The team leader (Leonardo Santi) was the **mediator of rational behaviour within the team**, but I may realize now, from some personal transcripts, that also young researchers assumed the same role within the team, with a great satisfaction by the team leader: *"OK, go on in your critical attitude towards your colleagues' work! We are becoming a strong scientific community!".* Literature was read in a careful way, in order to select the most useful results and reject other results (through a careful epistemic check (**ER**) based on theoretical and pragmatic elements).

Communication among us (**CR**) was always encouraged and supported by the team leader, and gradually I was enabled to enter the "**communication space**" of the team.

When the exploration was near to the end with many ideas at our disposal, the constructive phase started, according to a standard template, including the revision of the experimental design (**TR**), the evaluation through statistical methods (**TR, ER**) of the result, and its diffusion through an important journal (**CR**).

In this phase, arguments produced in the exploratory phase were carefully considered (**ER**) **and selected**; cogently argued conclusions were derived through a mixture of theoretically-based and factually-based inferences (**TR, ER**). But in some moments it was necessary to come back to exploration!

The resulting paper expressed a reasoned, critical position against the quick substitution of in-vivo tests with in-vitro tests (particularly the Ames test, very popular in that moment).

Parodi, S., Taningher, M., Boero, P. & Santi, L. (1982) Quantitative correlations amongst alkaline DNA fragmentation, DNA covalent binding, mutagenicity in the Ames test and carcinogenicity, for 21 compounds. Mutation Research, 93 (1), 1-24.

I must acknowledge now that at **that time my discovery of the apparently «a-scientific», haphazard initial ways of dealing with problems was a shock for me**, then I realized their **effectiveness**, and **now** I may realize that they were in the reality fully «rational», according to an intertwining of ER, TR and CR- inspired actions, intentionally oriented to create the condition for reaching, at the end, "solid" results. With the senior scientist who played the role of **mediator** of the rationality of the processes.

Strong similarities **now** emerge with the case of the **cognitive unity of theorems (when it is achieved): an exploration provides the elements for the construction of a scientific result!**

A final remark concerns **my critical attitude towards the STEM perspective**, when the role of mathematics (but also of other scientific disciplines, and technology) is reduced to their **pragmatic dimension of tools** to deal with interdisciplinary problems according to **linear, standard procedures**. This means to put aside those aspects of the disciplines that are not immediately productive in terms of preparation to act by using established tools and procedures. **The present prevailing pragmatic, utilitarian view of interdisciplinarity in STEM education looks IN CONTRAST with the necessity of developing a cultural transdisciplinary foundation** of the relationships between the disciplines in a general educational perspective, including how to deal with open problems, whose **effective treatment** demands the abovementioned **practice of rationality across different disciplinary domains.**

However also other theoretical elaborations might contribute **to clarify and frame transdisciplinarity**. At present, the Turin team leaded by Arzarello is working in the perspective of **Hintikka's Logic of Scientific Inquiry**, a framework different, but not in contrast and possibly **complementary** with Habermas' Rationality, as concerns **the dynamic interplay between ER and TR** hypothesised by Habermas.