## Conference in honor of Maria Alessandra Mariotti's career

Pisa, Dipartimento di Matematica,
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# Drawing tools for learning geometry: some aspects of the contribution of digital technologies

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Elements of the dynamic diagrams can be dragged in the presentation on line at https://go.cabri.com/pisa-2025

# Back to the nineties…

Mariotti, A.: 1992, 'Imagini e concetti in geometria', *L'Insegnamento Della Matematica e Delle Scienze Integrata* **15** ), 863–885.

Fischbein, E. The theory of figural *Educ Stud Math* **24** (1993).

… when I met Maria Alessandra for the first time. She was working on the theory of figural concepts with Fischbein.

And of course, she was interested in the nature of figures produced in a dynamic geometry environment (DGE), namely Cabri, as conceptual and figural aspects are closely intertwined in a DGE.

Maria Alessandra started working in classrooms with Cabri.

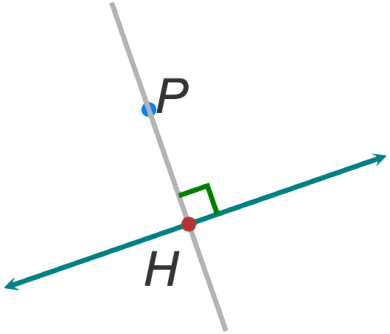
We had many opportunities of discussing at PME meetings, at a meeting in Oxford organized by Ros Sutherland, at the ICMI study on geometry in Catania (1995) and several times in Pisa…

… as well as in Hanoi at the ICMI study about digital technologies (2007)

# At the beginning of this century

Maria Alessandra completed a one-month research visit to Grenoble in 2000

What is the link with geometry ?



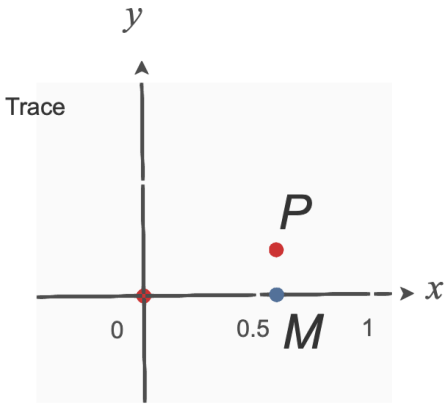

We started a common project about the learning of the notion of functional dependency and in particular of covariation.

Students' difficulties: grasping the idea of function as a relationship between variables, the one depending on the other one. They lack experience of functional relationship in a qualitative way.

The drag mode in dynamic geometry provides evidence of the distinction between direct motion and indirect motion.

In a Vygotskian perspective it may be considered as a *sign* referring to the idea of independent and dependent variables and of covariation between them.

# Tools as signs in the construction of meaning of function




Théorie des situations, médiation sémiotique et discussions collectives dans des séquences d'enseignement qui utilisent Cabri-géomètre et qui visent à l'apprentissage des notions de fonction et graphe de fonction / Rossana Falcade le 2006,

A difficulty for students is to link the global and pointwise aspects of the graph of a function.

The Trace tool in Cabri involves both a global meaning and a pointwise one. It may be considered as a *sign* referring to the mathematical notion of trajectory.

The graph of a function can be experienced as the trajectory of a moving point *P* with coordinates (x, f(x)) imagining that point *M* moves on the x-axis, (cf. text of Euler (1742))

Ph.D. thesis of Rossana Falcade linking the theories of didactic situations and semiotic mediation.

Teaching experiments were designed in France and in Italy with the aim of introducing students to the idea of function.

# Three years ago in 2022

Maria Alessandra and I were invited by Anne-Cécile Mathé to join her for presenting a report at the French national séminaire de didactique des mathématiques

Challenging topic: seeking solid findings in maths education research about geometry learning and teaching

Hard work raising many questions…

Eventually happy to make it!

# Message from Anne-Cécile Mathé

« Quelle chance d'avoir eu l'opportunité de travailler avec Maria-Alessandra ! Quel plaisir de voir son ouverture aux autres, sa clairvoyance dans les échanges scientifiques et sa bonne humeur toujours constante ! Quel richesse des échanges que nous avons pu avoir toutes les trois, Maria-Alessandra, Colette et moi, tant du point de vue scientifique que du point de vue humain ! Merci Maria-Alessandra pour toute cette bienveillance et curiosité de l'autre, qui font évidemment la marque des grand.es chercheure.s, et j'espère à très bientôt. »

"What a privilege it was to have had the opportunity to work with Maria Alessandra! It was such a pleasure to witness her openness towards others, her sharp insight during scientific discussions, and her constant good humour. The exchanges we shared were incredibly rich, both scientifically and on a personal level. Thank you, Maria-Alessandra, for your kindness and genuine curiosity about others, which are clearly the hallmarks of great researchers. I sincerely hope to see you again very soon."

# The graphic space of drawings

Two intertwined objectives of geometry curricula:

- geometry as modelling the perceived environment;
- geometry as a mathematical theory.

The graphic space of drawings may

- either model the world of concrete objects ;
- or represent the world of theoretical objects of geometry.

It is the interface between concrete and theoretical objects.

This explains the critical place of drawings in geometry teaching and the challenge geometry teaching is faced with:

guiding students in the move from action and interpretation of drawings controlled by perception to a control guided by theory.

# Use of tools in geometry teaching

"Tools matter: they stand between the user and the phenomenon to be modelled, and shape the activity structure." (Hoyles and Noss, 2003, p.341)

Producing drawings in geometry relies on drawing tools that incorporate geometric properties.

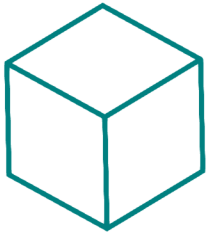
This is the reason why tools are used in geometry curricula.

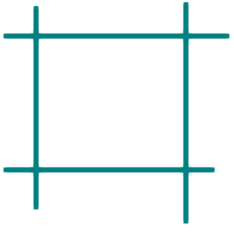
The underlying intention is to foster the use of geometrical properties by students and prepare them to have a theoretical control on drawings that is necessary in a proof activity.

R. Duval (2005) Les conditions cognitives de l'apprentissage de la géométrie. *Annales de didactique et sciences cognitives*, Vol. 10, 5-53

However constructing a proof requires not only interpreting the drawing associated to the problem but also transforming it: making more visible figural units (Duval 2005) and for this purpose drawing additional elements.

# Geometric visualization





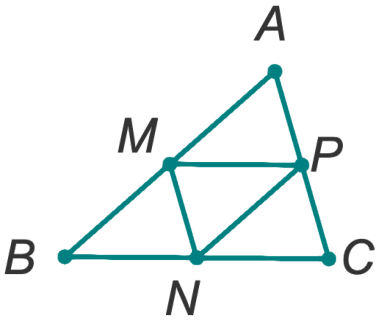
Duval claims that the major difficulty for students is to **see** geometrical properties in a drawing.

"Perception tends to impose a common way of seeing, which runs counter to the two approaches to figures used in mathematics education: one centred on constructing figures with instruments, the other one enriching them heuristically to reveal forms that are not immediately visible."

The cognitive mecanism underlying these two approaches is what Duval calls the *dimensional deconstruction*.

The mathematical way of visualizing a drawing is to break down it into figural units, not only 2D shapes but also 1D or 0D shapes. This decomposition is especially difficult, like in considering a side of a polygon as detached from its border. For students, there is a prevalence of 2D shapes over 1D shapes.

# A classical example: the inner triangle




PhD thesis of A.Lobo de Mesquita (1989)

Given : The sides of triangle *MNP* are parallel to the sides of triangle *ABC*.

Prove that point *N* is the midpoint of *BC*.

Students must isolate quadrilaterals *MBNP* and *NCPM* in order to prove that length *BN* = length *NC*.

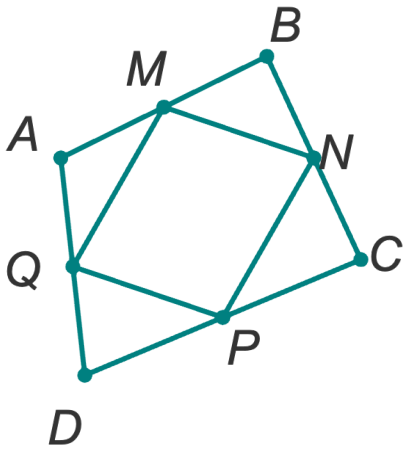
Double decomposition into figural units:

- into 2D shapes;
- identify the common segment *MP* in both borders of the 2D shapes.

R. Duval, M. Godin (2005) Les changements de regard nécessaires sur les figures. *Grand N* 76, 7-27

Making thicker the border of one quadrilateral, then of the other one on the drawing might detach each quadrilateral from the whole.

# A second classical example: Varignon




Given : *M* is the midpoint of *AB*, *N* is the midpoint of *BC*, *P* is the midpoint of *CD*, *Q* is the midpoint of *AD.*

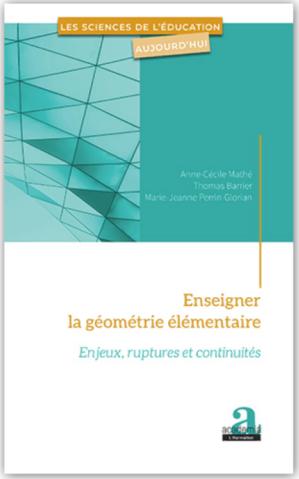
Prove that *MNPQ* is a parallelogram.

A proof requires considering the diagonals of quadrilateral *ABCD*. But they are not visible.

Proving is much easier if diagonals are drawn.

According to an implicit contract, students generally do not modify the drawing by adding an element. Very often the teacher has to prompt them.

# Goal of the geometry teaching




Mathé, Barrier, Perrin-Glorian 2020

Generally, the usual teaching of geometry places little emphasis on geometric reading and processing of a drawing before introducing the proof, in particular at primary level or beginning of junior high school.

However a type of situations requiring a geometric interpretation and a processing of a drawing was designed independently in France by two research works:

- black box situations in the DGE Cabri in the 90s ;
- situations of restoration of figures in a paper & pencil environment (Duval & Godin 2005).

# Cabri, a working environment on drawings

Two main ideas underpin the genesis of Cabri as expressed in its denomination. **Cabri** is an acronym for **Ca**hier de **br**ouillon **i**nteractif.

dialectical relationship

- Any activity in geometry, especially proof, requires exploring, experimenting on the drawing.
- Unlike paper & pencil drawings, Cabri drawings behave mathematically according to a theoretical model of geometry. This behaviour is particularly visible through the drag mode.

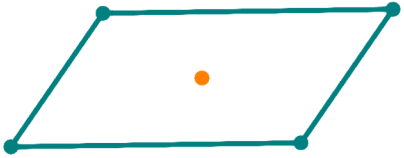
As a result, a Cabri drawing is not untouchable. It is recommended to work on Cabri drawings, to draw elements, to move its elements when solving a problem, because the behaviour of Cabri drawings is ruled according to geometry.

# Black box situations in Cabri

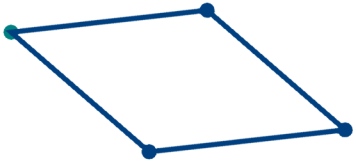
A dynamic drawing is given in Cabri. Some of its elements can be dragged.

The beginning of the drawing in another size and another orientation is also given.

Task of the students is to complete the reproduced drawing so that it **behaves** in the drag mode as the original one.



model



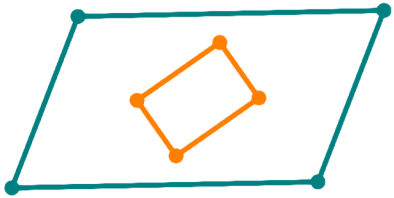
starting outline to complete

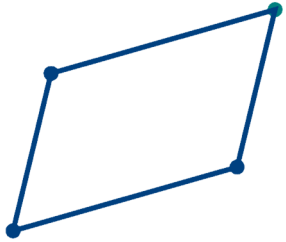
A prepared macro construction is used after completion of the task and shows the element to construct for validation of the student's work.

Charrière, P.-M. (1996). *Apprivoiser la géométrie avec Cabri-Géomètre*. Monographie du Centre informatique pédagogique, Genève

Black boxes for the learning of a scientific approach (PhD thesis of J.-J. Dahan, 2005)

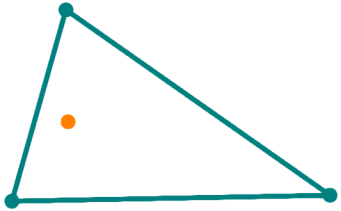
# Examples of black box situations in Cabri





given to a class of 13-14 year old students using regularly Cabri

Droite



in a teaching learning sequence with 15-16 year old students

# Geometry in action

In a black box situation, two kinds of actions support the analysis of relationships between the part to construct and the elements given in the drawing to complete.

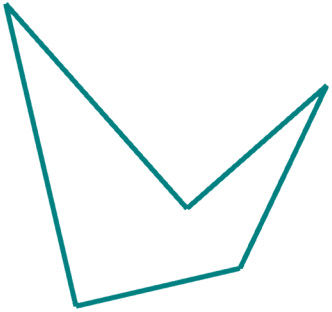
- construction of additional objects through Cabri commands for testing relationships.
- dragging the free elements in order to reveal the invariant relationships in the drag mode and dependencies between elements

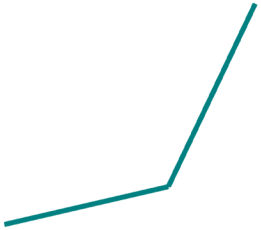
These situations were carried out with students having a regular use of Cabri at high school.

They were meant for learning how to identify in a drawing geometric relationships already taught, particularly using an interplay between commands and drag.

# Example of a restoration situation

A restoration task belongs the same kind of tasks as blackbox situations, but it takes place in a paper and pencil environment with concrete chosen tools.





Segment Set square

The solution is on a tracing paper given after completion of the task

Collective discussion in the class to gather the solving procedures.

Institutionnalisation of a formulation aimed at installing a first language describing geometric objects and relationships at stake.

# Restoration of figures at primary level

La géométrie des tracés, lien possible entre le dessin géométrique et la géométrie d'Euclide ?

A.-C. Mathé & M.-J. Perrin Glorian (2023)

Starting in the 2000s, teaching learning sequences at primary level were designed in a paper and pencil environment based on restoration of figures.

Their goal is to contribute to learn how to identify figural units by using a precise set of tools, introduced progressively: 2D templates, segment mediator, ruler without graduation, length protractor, angle protractor (in particular set square), compass.

Transformation of tools as external signs of theoretical control corresponding to axioms and theorems.

M. A. Mariotti 2002

Tools serve to draw with two different aims: for collecting information on the given drawing and to reproduce it.

Rules how to use the tools are given to students that match a practical axiomatic. Ex: in order to draw a straight line, one have to make the ruler passing through two points or along a segment.

# Developmental coordination disorder

As said above, drawings and tools play a critical role in the teaching and learning of geometry.

But drawing activities require considerable effort from students with developmental coordination disorder (DCD) (dyspraxia), as they often struggle with slowness and clumsiness, only to produce unsatisfactory results.

Drawing made by a sixth grade student
E. Petitfour 2017

They struggle to position the tools and to coordinate actions, such as holding the set square while moving the pencil to produce a drawing.

For a long time Cabri and now Cabri Express have been identified by occupational therapists as a valuable tool to help students with dyspraxia overcome the difficulties they face in learning geometry.

# Adaptation of Cabri Express for DCD students

DYSDYN

support of the French ministry of education

The current project DYSDYN gathers didacticians of mathematics, neuropsychologists and Cabrilog.

To make the classroom inclusive, functionalities have been added to Cabri Express, in order to adapt it for DCD students and allow them to have the same tasks as other students:

- keyboard control instead of mouse (DCD students' hand is tense on the mouse);
- import of photos, images, pdf files in Cabri Express, printing preserving the dimensions of the drawing;
- virtual tools, ruler with graduation, set square, compass and protractor.

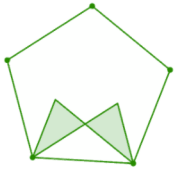
Cabri Express is freely accessible on line at

https://cabricloud.com/cabriexpress/primary/

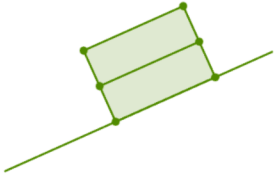
https://cabricloud.com/cabriexpress/secondary/

# Bringing together blackbox & restoration

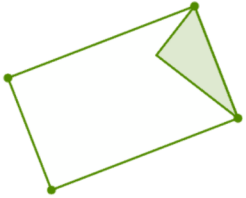
DYSDYN project brings together the blackbox and restoration situations.



A didactic engineering was designed and carried out in 11 classes last January at 4th and 5th grades for 5 weeks.



Introduction of the notions of midpoint, straight line, alignment, perpendicular line, parallel line, by means of drawing restoration & blackbox situations and collective discussions under the control of the teacher.



In each classroom

- part of the students in a paper & pencil environment
- the other part on Cabri Express

# Mille grazie a tutti

and to you, Maria-Alessandra, for your inspiring contribution to maths education research.

We are anxious to discover your new chapter(s)…