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Divide and conquer methods for functions of matrices with banded or hierarchical low-rank structure

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This talk is concerned with approximating matrix functions for banded matrices, hierarchically semiseparable matrices, and related structures [1]. We propose new divide-and-conquer methods – in the spirit of the divide-and-conquer algorithms developed in [2] for linear matrix equations – which exploit the fact that these matrices can be (recursively) decomposed as a sum $A = D + R$ of a block diagonal matrix D and a low-rank correction R. While the update $f(A) - f(D)$ often has low numerical rank and can be approximated via (rational) Krylov subspace projections [3, 4], the block diagonal part $f(D)$ is computed recursively for each diagonal block.

We present a convergence analysis that relates the accuracy attained by the algorithm with best polynomial or rational approximations of the function. For the special case of a banded matrix, we show that the divide-and-conquer method reduces to a much simpler algorithm, which proceeds by computing matrix functions of small submatrices of A. When only the trace or the diagonal of the matrix function is of interest, we demonstrate – in practice and in theory – that convergence can be faster.

Finally, we test the algorithms on a variety of matrices and functions; the numerical results demonstrate that, most of the time, the proposed methods outperform state-of-art techniques with respect to time consumption and offer a comparable accuracy.

References

[1] A. Cortinovis, D. Kressner, S. Massei, *Divide and conquer methods for functions of matrices with banded or hierarchical low-rank structure*, https://arxiv.org/abs/2107.04337.

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[4] B. Beckermann, A. Cortinovis, D. Kressner, M. Schweitzer, *Low-rank updates of matrix functions II: Rational Krylov methods*, SIAM Journal on Numerical Analysis (2021): 1325-1347.