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Non-intrusive model order reduction for cross-diffusion systems

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Non-intrusive reduced-order models (ROMs) are developed for parametrized cross-diffusion systems [1]

$$\begin{aligned} u_t^\mu &= d_u \nabla^2 u^\mu + d_{vu} \nabla^2 v^\mu + f(u^\mu, v^\mu; \mu), \quad (x,t) \in \Omega \times (0,T], \\ v_t^\mu &= d_{uv} \nabla^2 u^\mu + d_v \nabla^2 v^\mu + g(u^\mu, v^\mu; \mu), \quad (x,t) \in \Omega \times (0,T], \end{aligned} \tag{1}$$

where $\Omega \subset \mathbb{R}^d$ $(d = 2, 3)$ is the spatial domain, and $T > 0$ is the target time. The parameter dependent variables $u^\mu = u(x,t;\mu)$ and $v^\mu = v(x,t;\mu)$ represent chemical concentrations or population densities, ∇^2 is the Laplace operator, and $f(u^\mu, v^\mu; \mu)$, $g(u^\mu, v^\mu; \mu)$ are the nonlinear reaction terms. Exploiting the Kronecker structure in the finite-difference discretization of (1), the resulting system of ordinary differential equations (ODEs) are integrated in time with the semi-implicit Euler method in matrix or tensor form [2].

The ROMs are constructed using a two-level approach. In the first level, for each parameter value from a sample set of parameters, using higher-order singular value decomposition (HOSVD) [3, 4], the space-time coefficients of the ROMs are computed from the snapshot tensor data, which correspond to the core tensor of the truncated Tucker decomposition of the tensor. In the second level, applying standard singular value decomposition (SVD) to the matrix containing the reduced space-time coefficients for each parameter, the ROM basis is obtained in the training phase. In the test phase, reduced-order solutions are constructed for new parameter values using radial bases functions (RBF). The computational efficiency and accuracy of the ROMs are illustrated to predict the patterns of two examples of cross-diffusion systems of the form (1), the 2D Schnackenberg and 3D Brusselator equations.

References

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