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Operator-dependent prolongation and restriction for parameter-dependent multigrid methods using low-rank tensor formats

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We discuss the solution of parameter-dependent linear systems, i.e., $A(p)u(p) = f$, using parameter-dependent multigrid methods. Such a system arises, e.g., from a discretization of a PDE:

$$\begin{aligned} -\nabla \cdot (\sigma(x,p)\nabla u(x,p)) &= f(x) && \text{for } x \in \Omega, \\ u(x,p) &= 0 && \text{for } x \in \partial\Omega. \end{aligned} \tag{1}$$

In case of discontinuous $\sigma(x,p)$, e.g., if the parameters are jumping or random, $\nabla u(x,p)$ is discontinuous, too, cf. [1]. Therefore using standard linear interpolation for the prolongation and restriction, as in [2], is inaccurate and the convergence rate of a multigrid method declines.

In this talk, we motivate how to deal with these discontinuous $\sigma(x,p)$ in a parameter-dependent multigrid method.

To do so, we will recapitulate the convergence theory of parameter-dependent multigrid methods which we proved in [2]. This theory holds for arbitrary parameter-dependent problems. To achieve a data-sparse representation of the parameter-dependent linear system we recapitulate low-rank tensor formats. Our main question is then: How to deal with discontinuous $\sigma(x,p)$ in (1)?

We motivate the derivation of an operator-dependent prolongation and restriction based on block Gaussian elimination, cf. [3]. Numerical experiments using these operator-dependent prolongations and restrictions illustrate a fast convergence of low-rank tensor multigrid methods for discontinuous $\sigma(x,p)$.

References

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[3] A. Krechel, K. Stüben, *Operator Dependent Interpolation in Algebraic Multigrid*, in Lecture Notes in Computational Science and Engineering, Springer Berlin Heidelberg, 1998.