Matrix Equations and Tensor Techniques
IX
Perugia, September 9-10, 2021

Model reduction and utilization of the reduced basis for aggregation kinetic equations

Ivan Timokhin *Sergey A. Matveev* Alexander Smirnov Eugene Tyrtyshnikov

Marchuk Institute of Numerical Mathematics RAS, Gubkin st 8, 119333, Moscow

In this work we apply Proper Orthogonal Decomposition (POD) for model reduction in application to kinetic equations of irreversible aggregation process with source of monomers:

$$\frac{dn_k}{dt} = J_k + \frac{1}{2}\sum_{i+j=k} C_{i,j} n_i n_j - n_k \sum_{j=1}^{N} C_{j\,k} n_j, \qquad k = \overline{1, N} \tag{1}$$

We assume the existence of an orthonormal basis, gathered as columns of a matrix $V \in \mathbb{R}^{N \times R}$, such that

$$\left\| n(t) - VV^T n(t) \right\| \ll \| n(t) \|, \tag{2}$$

where $n(t)$ is the solution of the original system and further introduce the reduced solution

$$x(t) \equiv V^T n(t), \qquad x(t) \in \mathbb{R}^R \tag{3}$$

which can be evaluated within $O(R^3)$ operations instead of $O(N \log N)$ required for the original system. In case $R \ll N$ evaluation of the reduced solution requires much less operations and depends only on the dimensionality of the basis. Final reconstruction of the original solution can be done as

$$n(t) \approx \tilde{n}(t) = V \tilde{x}(t). \tag{4}$$

Thus, the main problem is choice of algorithm for construction of the target basis allowing to perform the reduction of dimensionality. In our work we show that POD exploiting the method of "snapshots" allows to obtain the basis and in our experiments we show that $R \ll N$.

All in all, we also show that utilization of such basis allows to perform a speedup of compuations for aggregation kinetic equations without significant loss of accuracy of the solutions. At the same time, we also demonstrate problematic sides of the chosen approach – the control of the precision of the reduced solution seems to be not an easy task due to the nonlinearity of both method and model and requires additional studies in future.

This work has been supported by Russian Science Foundation Project (21-71-10072).

References

[1] I. V. Timokhin, E. E. Tyrtyshnikov, A. P. Smirnov, *Model reduction in Smoluchowski-type equations*, arXiv preprint arXiv:2008.09439 (2020)

[2] I. V. Timokhin, E. E. Tyrtyshnikov, A. P. Smirnov, *Method for Reduced Basis Discovery in Nonstationary Problems*, Doklady Mathematics, Vol. 103 No. 2 (2021)