

Restarts for matrix Laplace transforms

Saturday, 11 June 2022 12:45 (30 minutes)

Let $A \in \mathbb{C}^{n \times n}$ be a matrix, $v \in \mathbb{C}^n$ a vector, and $f : D \subseteq \mathbb{C} \rightarrow \mathbb{C}$ a function defined on the spectrum of A . We consider the task of computing $f(A)v$, the action of the matrix function $f(A)$ on the vector v . This task arises in many applications and it is a viable alternative to computing $f(A)$ directly, which is impossible if A is large and sparse.

In this talk we consider the Arnoldi approximation, a Krylov subspace extraction for $f(A)v$. Since a new vector needs to be generated and stored in every iteration, one is often forced to rely on restart algorithms which are either not efficient or not stable or only applicable to restricted classes of functions. This holds already to some extent in the symmetric case, because of storage restrictions, and is very prominent in the non-symmetric case due to the increasing cost of orthogonalizations.

We present a new representation of the error of the Arnoldi iterates if the matrix function f can be represented as a Laplace transform

$f(t) = \int_0^\infty g(s)e^{-st} ds$, and develop a restarted algorithm for computing $f(A)v$ based on this error representation. The error representation is based on the error representation of the Arnoldi iterates, and it generalizes earlier work of some of the author on quadrature-based restarts for Stieltjes matrix functions. The error representation is based on the error representation of the Arnoldi iterates, and it generalizes earlier work of some of the author on quadrature-based restarts for Stieltjes matrix functions. The error representation is based on the error representation of the Arnoldi iterates, and it generalizes earlier work of some of the author on quadrature-based restarts for Stieltjes matrix functions.

We will present several numerical experiments illustrating the efficiency of our approach, including fractional diffusion on graphs and the matrix Gamma function.

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