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On non-Hermitian positive (semi)definite linear algebraic systems arising from dHDAE systems

Saturday, 11 June 2022 09:15 (30 minutes)

It is well known that every matrix $A \in \mathbb{C}^{n \times n}$ can be split into its Hermitian and skew-Hermitian parts, i.e., $A = H+S, \quad H = \frac{1}{2}(A+A^*), \quad S = \frac{1}{2}(A-A^*)$,*sothat*H = H^{}*andS*=-S^{\}.*Thissimple*, *yetfundamentalobservationhasmany*
= H + S*lieinoronthesmallestrectanglewithsidesparalleltotherealandimaginaryaxesthatcontainsalleigenvaluesof*H*andof*S.
Hermitian)*matricesAaresometimescalledpositiverealorpositivestable*, *butwecall*A = H + S*positive*(*semi*)*definiteif*H*hasthe*

In the first part of this talk we will discuss an important class of practically relevant applications where the splitting $A = H+S$*occursnaturallyandhasaphysicalmeaning*.*Theseapplicationsarisefromenergy*−
basedmodelingusingdifferentialalgebraicequation(*DAE*)*systemsindissipativeHamiltonian*(*dH*)*form*, *orshortl*

In the second part we will study the linear algebraic systems arising in this context, and their solution by Krylov subspace methods. The (non-Hermitian) matrices A that occur are positive definite or positive semidefinite. In the positive definite case we can solve the linear algebraic systems using Krylov subspace methods based on efficient three-term recurrences. Such methods were already derived by Widlund [6] (based on earlier work of Concus and Golub [3]) and Rapoport [5] in the late 1970s. These methods are not widely used or known, and we will therefore summarize the most important facts about their implementation and their mathematical properties. The semidefinite case can be challenging and requires additional techniques to identify and deal with the "singular part"of the matrix, while the "positive definite part"can still be treated with the three-term recurrence methods. We will illustrate the performance of the iterative methods and compare them with (preconditioned) GMRES on several computed examples.

The talk is based on joint work with Candan Güdücü, Volker Mehrmann, and Daniel B. Szyld [4].

References

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