

# Sobolev regularity of flows associated to vector fields with exponential or sub-exponential summability

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We are concerned with the Sobolev regularity of a flow  $X : I \times I \times \Omega \rightarrow \Omega$  associated to a non-smooth vector field  $b : I \times \Omega \rightarrow \Omega$ , i.e. the solution of the Cauchy problem

$$\begin{cases} \partial_t X(t, s, x) = b(t, X(t, s, x)) & t, s \in I, x \in \Omega, \\ X(s, x) = x \end{cases}$$

where  $\Omega \subset \mathbb{R}^n$  is a given open domain and  $I \subset \mathbb{R}$  is a given interval. We are going to discuss assumptions on vector field  $b$  in order that (P) is well-posed, that is, if it admits existence and uniqueness. Moreover we will focus on the Sobolev regularity of the associated flow  $X$ , that is, whether, for a given  $p \geq 1$ ,  $X(t, s, \cdot) \in W_{loc}^{1,p}(\Omega_{(t,s)}, \mathbb{R}^n)$  for given  $t, s \in I$ , where  $\Omega_{(t,s)}$  denotes the open set of  $x \in \Omega$  such that the path starting at  $x$  at time  $s$  can be extended until time  $t$ . We will review some well-known results in this topic and we will present some new results which are part of a joint work with L. Ambrosio and S. Nicolussi Golo (Jyväskylä). Eventually an application will be given to the Bernstein problem for area-minimizing intrinsic graphs in the sub-Riemannian first Heisenberg group, which is part of a joint work with S. Nicolussi Golo and Mattia Vedovato (Trento) still in progress.

**Presenter:** SERRA CASSANO, Francesco (Università di Trento)