

Sobolev regularity of flows associated to vector fields with exponential or sub-exponential summability

Wednesday, 14 June 2023 14:30 (1 hour)

We are concerned with the Sobolev regularity of a flow $X : I \times I \times \Omega \rightarrow \Omega$ associated to a non-smooth vector field $b : I \times \Omega \rightarrow \Omega$, i.e. the solution of the Cauchy problem

$$\begin{cases} \partial_t X(t, s, x) = b(t, X(t, s, x)) & t, s \in I, x \in \Omega, \\ X(s, x) = x \end{cases}$$

where $\Omega \subset \mathbb{R}^n$ is a given open domain and $I \subset \mathbb{R}$ is a given interval. We are going to discuss assumptions on vector field b in order that (P) is well-posed, that is, if it admits existence and uniqueness. Moreover we will focus on the Sobolev regularity of the associated flow X , that is, whether, for a given $p \geq 1$, $X(t, s, \cdot) \in W_{loc}^{1,p}(\Omega_{(t,s)}, \mathbb{R}^n)$ for given $t, s \in I$, where $\Omega_{(t,s)}$ denotes the open set of $x \in \Omega$ such that the path starting at x at time s can be extended until time t . We will review some well-known results in this topic and we will present some new results which are part of a joint work with L. Ambrosio and S. Nicolussi Golo (Jyväskylä). Eventually an application will be given to the Bernstein problem for area-minimizing intrinsic graphs in the sub-Riemannian first Heisenberg group, which is part of a joint work with S. Nicolussi Golo and Mattia Vedovato (Trento) still in progress.

Presenter: SERRA CASSANO, Francesco (Università di Trento)